

## **An NLP Approach for Evolution of Heat Exchanger Networks Designed by Pinch Technology**

*E. Rezaei\*, S. Shafiei*

*Faculty of Chemical Engineering, Sahand University of Technology, Tabriz, Iran.*

### **Abstract**

*Common methods to design heat exchanger networks (HENs) by pinch technology usually need an evolutionary step to reduce the number of heat transfer units. This step is called loop breaking and is based on the removal of exchangers that impose minimum increase on utility consumption. Loops identification and breaking is a tedious task and becomes more complicated in large networks. This paper presents a rapid nonlinear programming (NLP) formulation for the evolution of HENs in which loop identification is not required. The objective of the NLP is the minimization of HENs annual cost, which is not considered in current methods. In this method a search is done to find the best units elimination of which improves HENs annual cost. The search continues until the minimum number of units (MNU) is achieved and the exchangers that must be removed from the network are specified. The method was applied to some networks reported in the literature and better results were obtained. Also, the convergence of the presented method is very fast and it can be applied to different networks designed by pinch technology.*

**Keywords:** *Heat exchanger networks (HENs), Minimum number of units (MNU), Loop breaking, Nonlinear programming (NLP)*

### **1- Introduction**

There are three major methods for heat exchanger network synthesis. The first is pinch technology and is based on thermodynamic concepts that have been introduced by Linnhoff and Flower [1] and Linnhoff and Hindmarsh [2]. The second belongs to optimization methods and the minimization of total annual cost of networks by mathematical programming that has been introduced by Ciric and Floudas [3] and Yee and Grossmann [4]. The latter is the methods that combine the above concepts proposed by

Zhu and Nie [5].

HEN synthesis by pinch technology needs post analysis to reach the minimum number of units (MNU) target. This step causes HEN annual cost to decrease and it depends on the identification of loops within a network. This is a complicated task, particularly in large networks. Su and Motard [6] have used the graph theory for finding loops and Incidence matrix has been introduced by Pethe et al. [7] to identify network loops. In fact, each loop contains an additional unit and its breaking leads to eliminate one exchanger from the

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\* Corresponding author: E-mail: e\_rezaei@sut.ac.ir

network. After the breakage of each loop, a path relaxation is used to avoid crossing heat from pinch and restore feasible heat transfer within the network. So this relaxation imposes additional utility consumption on the network. Thus the exchanger, elimination of which imposes the smallest additional utility load should be removed.

Linnhoff and Hindmarsh [2] suggested some heuristics to select the best exchanger to be removed from each loop. As these rules are not correct in all cases, Trivedi et al. [8] have developed a systematic method called LONITA<sup>1</sup> for loop breaking. Although their method is efficient, the identification of loops is still inevitable. Zhu et al. [9] developed a method which is based on the use of the lower bound of energy penalty incurred by deleting a match. This lower bound is determined by calculating a redistribution of minimum approach temperature approaches after removing a match from the HEN.

Mathematical programming was first used by Jezowski et al. [10] for network evolution. The problem was formulated as mixed integer linear programming (MILP) and cost evaluation was not inserted in their formulation. On the other hand, their objective was to minimize the number of units simultaneously with utility loads. The most important point of this method is that it does not require loop identification. In fact in their work, area calculations are removed from formulation and the problem is defined as an MILP instead of MINLP, so finding the best solution is not guaranteed.

This paper presents an automated method in which cost of network is utilized as a criteria for removing exchangers. So the problem is formulated as an NLP and its objective is minimization of HEN annual cost which is composed of area and energy costs. In this method each exchanger is removed first and the cost of the HEN is calculated after its

removal. Hence the best choices for elimination can be identified after some iterations. In the next sections methodology, HEN representation, NLP formulation, case studies and conclusion follow.

## 2- Methodology

The first thing that must be determined is finding the minimum number of units (MNU) in a network. The overall unit target is given by:

$$N_{U,\min} = N_s - 1 \quad (1)$$

Where  $N_s$  is the number of streams (both process and utility).

The target for minimum number of units in an MER network is defined by:

$$N_{U,\text{MER}} = \sum_{j=1}^{P+1} (N_s - 1)_j \quad (2)$$

Where  $P$  is the number of pinches,  $i$  refers to  $i^{\text{th}}$  sub problem after division at pinches. The number of loops in any network is determined by:

$$N_l = N_{U,\text{MER}} - N_{U,\min} \quad (3)$$

This method initially finds the first unit, elimination of which causes minimum annual cost. This procedure is repeated  $N_l$  times and the best unit for removing is specified after each iteration. It is obvious that if removal of any unit does not improve HEN annual cost in  $i^{\text{th}}$  iteration, the search is complete.

It has been proved by Suaysompol and Wood [11], Jezowski [12] and Floudas et al. [13] that it is unnecessary to set EMAT<sup>2</sup> equal to HRAT<sup>3</sup> and the following inequality must be used for these parameters:

$$\text{EMAT} \leq \text{HRAT} \quad (4)$$

1 - LOP Network Interaction and load Transfer Analysis

2 - Exchanger Minimum Approach Temperature  
3 - Heat Recovery Approach Temperature

This inequality may cross some heat from pinch and create networks with lower costs. Thus in this work EMAT is relaxed and the best EMAT is searched in NLP formulation. In fact, the NLP determines that the value of EMAT must be equal or less than HRAT. The overall algorithm is shown in Fig. 1.

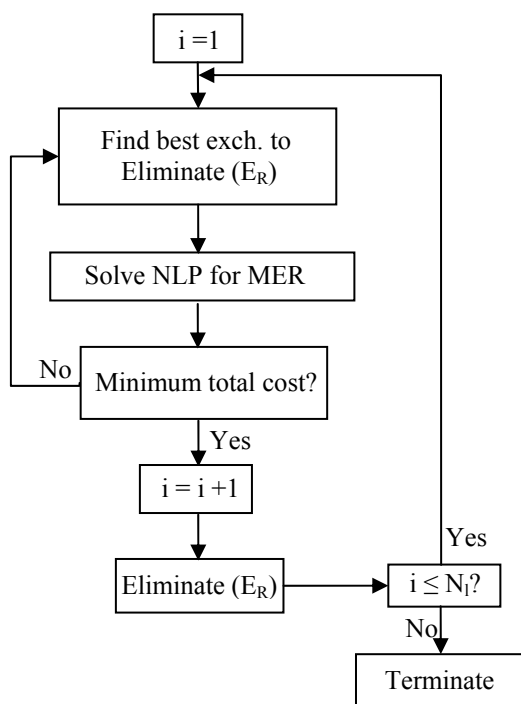


Figure 1. Overall algorithm to reach MNU

### 3- Network representation

In the present method a HEN is treated as a sequence of stages and each stage includes the addresses of some exchangers. For addressing the location of exchangers the node representation is used like Fig. 2, in which the number of splitters and their branches can be set manually in each stage. This kind of addressing is usual and has been used by some researchers like Bochenek and Jezowski [14] and Zhu and Asante [15].

When two nodes are selected, an exchanger is defined between them. Consider a HEN shown in Fig. 3 with three exchangers. An exchanger address matrix (EAM) is defined to show the location of exchangers in the network like the following matrix for Fig. 3.

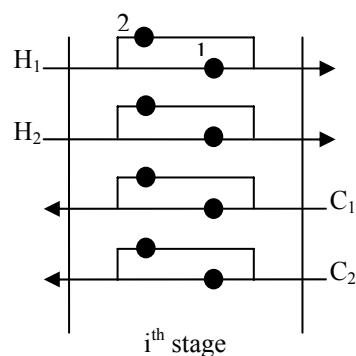


Figure 2. Nodes in each stage with 2 branches in each splitter

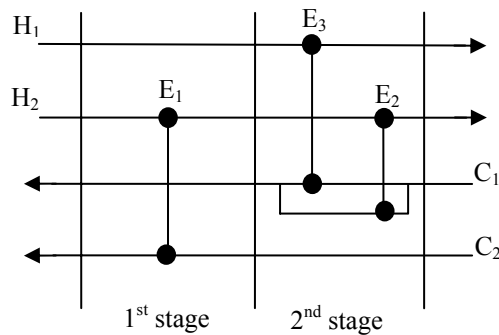


Figure 3. An HEN with three exchangers

$$EAM = \begin{bmatrix} E_1 : 2 & 1 & 2 & 1 & 1 \\ E_2 : 2 & 1 & 1 & 2 & 2 \\ E_3 : 1 & 1 & 1 & 1 & 2 \end{bmatrix}$$

In this matrix each row is an address of an exchanger.

If splitting occurs in a stage, nodes of the splitter are numbered from 1 to the number of branches, otherwise the number of each node will be 1. In the EAM the first column is the hot stream number, the 2<sup>nd</sup> is the node number of the hot stream. The 3<sup>rd</sup> and the 4<sup>th</sup> are similar numbers for cold streams. The 5<sup>th</sup> column represents the stage number.

### 4- NLP formulation

Lewin et al. [16], [17] have used an NLP formulation for maximum energy recovery

(MER) and the present formulation is based on their method. In this approach area calculations are not considered explicitly in the formulation and a penalty term is added to reduce costs as much as possible. In fact this term modifies the objective function and relaxes some exchangers from pinching at EMAT. The objective function is:

$$\text{Maximize } \sum_{i=1}^{\text{no. of exch.}} X_i + \left( \sum_{i=1}^{2(\text{no. of exch.})} \Delta T \right) / \text{S.F.} \quad (5)$$

Where  $X_i$  is the load of exchangers and  $\Delta T$  is approach temperatures in the hot or cold end of the exchangers. S.F. is a scaling factor and must be large enough to ensure that the penalty term do not affect the main objective which is maximum energy recovery.

Constraints of this NLP are:

- Energy balance for each exchanger on hot and cold streams. (nonlinear if splitting occurs)
- Energy balance for hot and cold utilities. Heaters and coolers are included in the formulation and if they are not needed, the NLP sets their loads to zero. (Linear). This type of formulation helps the optimization to consider more possibilities for HEN evolution.
- Mass balance for splitters. (Linear)
- Monotonic decrease of temperatures on streams. (Linear)
- Hot and cold end approach temperatures must be equal or greater than EMAT in each exchanger including utility exchangers. (Linear)
- Energy balance at mixing points. (Linear)

In this formulation split ratios ( $y$ ) and minimum approach temperatures are not optimized simultaneously with exchanger heat loads. Instead, an inner loop is utilized for finding the best  $y$  and EMAT. In this loop the problem is converted to a modified linear programming for finding MER. Therefore the NLP is converted to a search for  $y$  and EMAT and an LP for MER due to

elimination of nonlinear terms by knowing split ratios.

The algorithm of the NLP is shown in Fig. 4.

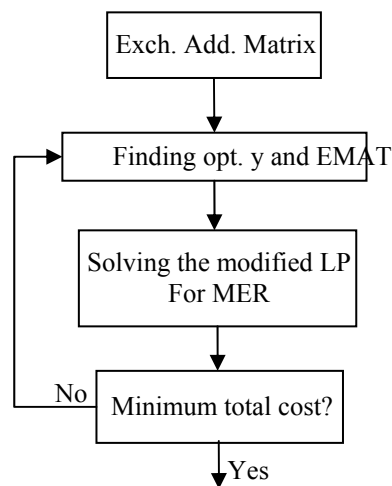


Figure 4. The NLP algorithm

Care must be taken for small split ratios because they cause the LP to be ill conditioned. So split ratios are bounded from 0.1 to 0.9 and the search range for EMAT is set to  $[0.1, 30^\circ\text{C}]$ .

## 5- Case studies

Three case studies are solved by MATLAB codes. The first is example 4S1 of Shenoy [18], the second and third belong to Jezowski et al. [10]. The application of the present method yields better solutions than those reported in the literature. For all these problems, counter current heat exchangers were considered. In the following examples,  $y$  refers to the split ratio of the top branch.

### 5-1- Case 1

This case is solved by Shenoy [18]. The original network has six exchangers and one splitter with an annual cost of 245140 \$/yr and  $\text{EMAT}=\text{HRAT}=13^\circ\text{C}$ . Original HEN is indicated in Fig. 5 with 2 exchangers more than MNU. Also, Table 1 shows the data for this problem. In this figure the underlined numbers are heat loads.

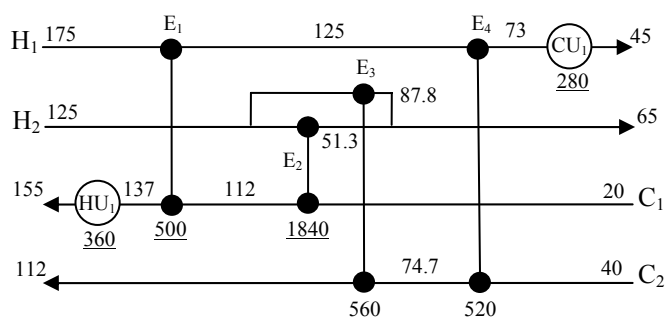


Figure 5. Original HEN for case 1

Table 1. Data for case 1

Streams	T <sup>in</sup> (°C)	T <sup>out</sup> (°C)	FC <sub>p</sub> (kW/°C)	Cost (\$/kW/yr)
H <sub>1</sub>	175	45	10	
H <sub>2</sub>	125	65	40	
C <sub>1</sub>	20	155	20	
C <sub>2</sub>	40	112	15	
Steam	180	179		120
Water	15	25		10

U=0.1 kW/m<sup>2</sup>K for all exchangers  
 Cost (\$) = 30000+750A<sup>0.81</sup> for all exchangers, A in m<sup>2</sup>  
 Plant life time: 5 yr, Rate of interest: 10%  
 LMTD is used for area calculations

The results after the first search are shown in Table 2. As can be seen, the elimination of exchanger 4 improves HEN annual cost.

Table 2. Results after first search

Elimination	HEN cost (\$/yr)	Split ratio	EMAT (°C)
E <sub>1</sub>	256747	1	11.11
E <sub>2</sub>	311085	No split	5
E <sub>3</sub>	245271	No split	5
E <sub>4</sub>	239797	0.47	7.58

After elimination of the fourth exchanger, the search continues to find the next exchanger and results are shown in Table 3.

No improvement was obtained with respect to 239797 \$/yr and the search is terminated. Shenoy [18] has eliminated unit 4, but has restored EMAT=HRAT=13°C. This causes

the cost to increase to 244580 \$/yr, which is 2% greater than our solution. This difference is created because of fixing the value of EMAT to 13°C. The final network is shown in Fig. 6.

Table 3. Results after second search

Elimination	HEN cost (\$/yr)	Split ratio	EMAT (°C)
E <sub>1</sub>	329624	0.734	15
E <sub>2</sub>	311085	No split	10
E <sub>3</sub>	349730	No split	9.2

### 5-2- Case 2

This example has been solved by Jezowski et al. [10]. As no cost data was needed in their formulation, the cost data of case 1 is used for the estimation of area and utility costs. The original HEN has a cost of 96803\$/yr and is shown in Fig. 7.

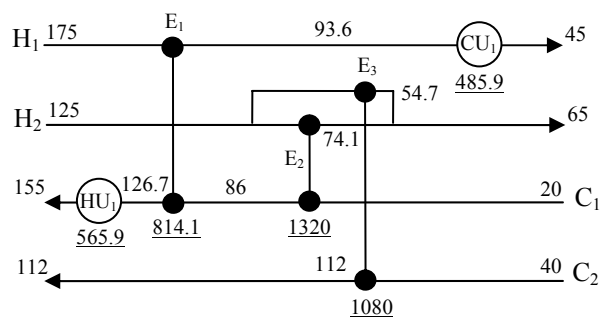


Figure 6. Final HEN for Case 1

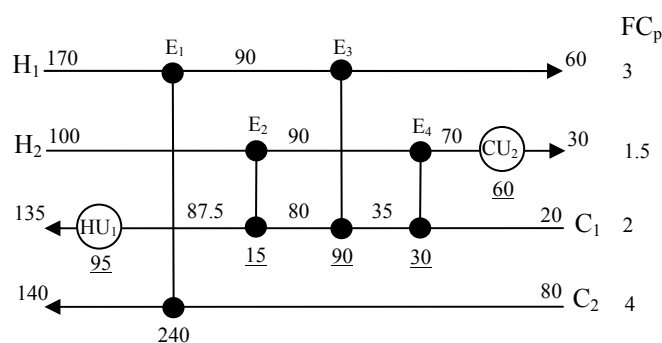


Figure 7. Original HEN for case 2

Jezowski et al. [10] have eliminated only exchanger 4 from the network and have fixed  $EMAT=HRAT=10^{\circ}C$ . In this way the cost of HEN can be decreased to 86494 \$/yr. The present method suggests the removal of unit 4 and 3 which improves HEN annual cost to 78671 \$/yr. This difference indicates the important role of the cost calculation for unit

reduction. On the other hand, utility consumption and unit numbers are not enough to be considered in the optimization. The best way for HEN evolution is the NLP formulation which minimizes the total cost of the network. The best network for this case is shown in Fig. 8.

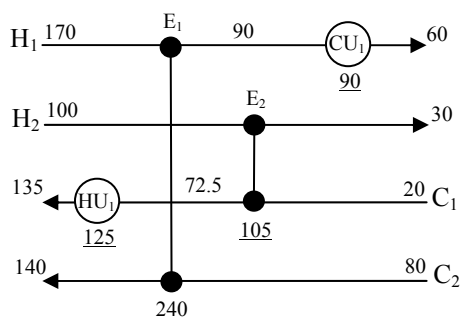
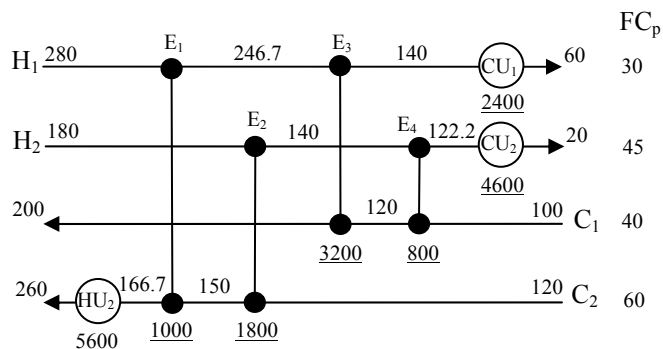


Figure 8. Best HEN for case 2

**5-3- Case 3**

The initial network of this case is shown in Fig. 9 with a cost of 1100595 \$/yr and  $EMAT=HRAT=20^{\circ}C$ . As for case 2, the

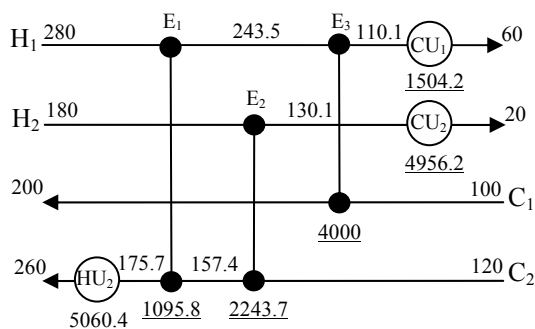
cost data of example 1 is used. Inlet and outlet temperatures of cooling water and steam are set to  $[5-10^{\circ}C]$  and  $[300,300^{\circ}C]$  respectively.



**Figure 9.** Initial HEN for case 3

Jezowski et al. [10] have restored  $EMAT=HRAT=20^{\circ}C$  and decreased the cost to 1094447 \$/yr by removing unit 4. Although the present method proposes the elimination of exchanger 4, it reduces the

value of  $EMAT$  to  $10.14^{\circ}C$ . The optimal network is indicated in Fig. 10 with an annual cost of 1060071 \$/yr which is lower than that obtained by Jezowski et al. [10].



**Figure 10.** Optimal HEN for case 3

**6- Conclusion**

In this paper an optimization method is presented to help pinch technology to synthesize optimal HENs. This method is based on the minimization of the total annual cost by removing exchangers from networks.

In fact, it considers total costs instead of the energy penalties which are usually used in current methods as a criteria for eliminating exchangers.

Results show that it is necessary to consider the trade-off between area and energy costs

for unit reduction. On the other hand, if this trade-off is not considered, optimal HENs cannot be designed with common approaches. Also, as it was shown, setting  $EMAT \leq HRAT$  may improve HENs annual cost in this evolutionary step. Therefore methods like the pinch design method (PDM) or any approach which is based on pinch technology can be enriched with an optimization method to reach the most efficient networks.

The presented method is faster than commercial NLP solvers. This is because the NLP formulation is converted to a modified LP for MER and a search for finding the best EMAT and split ratios. Also, this method does not require the identification of loops which is a tedious task in medium and large scale networks. Due to these advantages, this method can be easily applied to any networks designed by pinch technology.

### Nomenclature

$C_i$	$i^{\text{th}}$ cold stream, $i=1 \dots, n_C$
$CU_i$	$i^{\text{th}}$ cold utility exchanger, $i=1 \dots, n_H$
$E_i$	$i^{\text{th}}$ exchanger
$FC_p$	heat capacity flow rate
$H_i$	$i^{\text{th}}$ hot stream, $i=1 \dots, n_H$
$HU_i$	$i^{\text{th}}$ hot utility exchanger, $i=1 \dots, n_C$
LMTD	logarithmic mean temperature difference
$N_l$	number of independent loops
$N_s$	number of streams (both process and utility)
$N_{U, \text{MER}}$	minimum number of units
$N_{U, \text{Min}}$	overall unit target
$U$	overall heat transfer coefficient
$X_i$	heat load of $i^{\text{th}}$ exchanger
$y$	split ratio
$\Delta T$	approach temperature in hot or cold end of an exchanger

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