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An Efficient Coupled Genetic Algorithm-NLP Method for Heat Exchanger Network Synthesis

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Abstract
Synthesis of heat exchanger networks (HENs) is inherently a mixed integer and nonlinear programming (MINLP) problem. Solving such problems leads to difficulties in the optimization of continuous and binary variables. This paper presents a new efficient and robust method in which structural parameters are optimized by genetic algorithm (G.A.) and continuous variables are handled due to a modified objective function for maximum energy recovery (MER). Node representation is used for addressing the exchangers and networks are considered as a sequence of genes. Each gene consists of nodes for generating different structures within a network. Results show that this method may find new or near optimal solutions with a less than 2% increase in Hen annual costs.

Keywords*: Heat exchanger networks (HENs), Optimization, Genetic Algorithm (G.A.), NLP formulation*

1- Introduction
There are three major methods for heat exchanger network synthesis. The first is pinch technology and is based on thermodynamic concepts that have been introduced by Linnhoff and Flower [1] and Linnhoff and Hindmarsh [2]. A good review on this technology is Shenoy's book [3]. The second one belongs to optimization methods and the minimization of the total annual cost of networks by mathematical programming that have been proposed by Ciric and Floudas [4] and Yee and Grossmann [5]. A summary of the history of these methods has been collected by Floudas [6]. In recent years, trends in mathematical methods were to simplify MINLP formula-tion and attain a global solution like the MILP synthesis method [7], graph theory [8] and global optimization [9]. The last one is methods that combine the above concepts proposed by Zhu and Nie [10].

In mathematical programming methods the problem is defined as an MINLP problem and is solved by deterministic, stochastic or coupling them. Deterministic methods like GBD, OA and etc failed to converge due to the mixed nature of binary and continuous variables.

Stochastic methods like simulated annealing have been applied (S.A.) by Athier et al. [11], [12], genetic algorithm (G.A.) by Lewin

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et al. [13], [14], and tabu search (T.S.) by Lin and Miller [15].

Athier et al. [11], [12] have used a coupled S.A.-NLP method in which S.A. controls structural optimization and SQP optimizes continuous parameters. Anyway, S.A. cannot reach global structure and the writers had problems in the convergence of their NLP. A G.A. and S.A. coupled method has been used for the optimization of both binary and continuous variables by Yu et al. [16] to avoid trapping in local optima. As it is known, handling the constraints in the G.A. needs special aspects, and convergence in continuous parameters leads to many iterations. Therefore, due to the discrete nature of the G.A., it seems that the it is very useful and efficient for structural optimization.

In this paper, G.A. is used for structural parameters, while the fitness of each structure is specified by a modified NLP formulation that is based on maximum energy recovery (MER), which has been described by Lewin et al. [13], [14].

In the following sections the new representation of the HEN structure is mentioned and G.A. operators and NLP formulation are presented. Some case studies are then studied and the results are compared with those reported in the literature.

2- Problem definition

The problem can be defined as follows:

A set of hot and cold streams with their inlet and outlet temperatures are given. Also heat capacity flow rates and heat transfer coefficients of the streams are known. Hot and cold utility are available as external sources for the cooling and heating of the process streams. The objective is to design a network with the minimum total annual cost which is determined by the summation of area and utility costs.

3- Representation of network

In the present method a HEN is treated as a chromosome and exchangers within it are considered as a sequence of genes. Therefore each gene includes the address of an exchanger. For addressing the location of exchangers the node representation is used like Fig. 1, in which the number of splitters and their branches can be set manually in each gene. This kind of addressing is usual and has been used by some researchers like Bochenek and Jezowski [17] and Zhu and Asante [18].

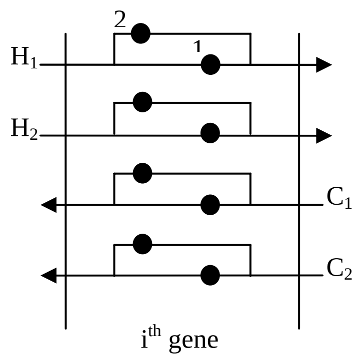

Figure 1. Nodes in each gene with 2 branches in each splitter

When two nodes are selected, an exchanger is defined between those nodes. The number of genes in each network depends on the size of the problem and varies in different case studies. Consider an HEN shown in Fig. 2 with three exchangers.

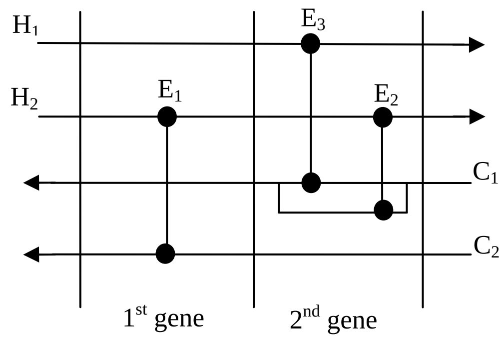

Figure 2. An HEN with three exchangers

An exchanger address matrix (EAM) is proposed for showing the location of exchangers in the network like the following matrix for Fig. 2. in which each row is an address of an exchanger.

$$\mathrm{EAM} = \begin{bmatrix} E_1 : 2\ 1\ 2\ 1\ 1 \\ E_2 : 2\ 1\ 1\ 2\ 2 \\ E_3 : 1\ 1\ 1\ 1\ 2 \end{bmatrix}$$

If splitting occurs in a gene, the nodes of the splitter are numbered from 1 to the number of branches, otherwise the number of each node will be 1. In the EAM the first column is the hot stream number, the 2nd is the node number of the hot stream. The 3rd and the 4th are similar numbers for cold streams. The 5th column represents the gene number.

This representation is specially suited for G.A. operators in order to create feasible structures which will be described in the next section.

4- G.A. operators

Three operators are used for this evolutionary algorithm, reproduction, crossover and mutation.

a) Reproduction: In each population some structures are exactly copied to the next generation due to the survival rate which is set at 40-50%. Selection of these chromosomes is determined by their relative fitness and roulette wheel. For more robustness of the algorithm elitism is added to save the best solution in each population. The number of structures in the initial population depends on the size of the problem. In small scale examples 20 chromosomes are sufficient to reach the best solution. Note that the initial population is produced randomly. It is clear that the number of iterations is proportional to the size of the initial population and chromosome length. So 40-50 iterations are employed to reach stopping criteria of the algorithm.
b) Crossover: For combining the genetic materials, single point crossover is used. Two parents are selected by roulette wheel and a random gene number is generated to decompose the parents into four parts. These parts can combine together to produce offspring. In this article the rate of crossover is 50-60%.
c) Mutation: The definition of mutation and its rate plays an important role in the convergence of the algorithm. Because the NLP formulation is able to set the heat load of some exchangers to zero, mutation does not eliminate exchangers from the network and only changes the address of exchangers in genes. This operator removes the whole exchangers in a gene and considers new random addresses within the gene. In this way a splitter may be removed and an exchanger may be replaced or vise versa. The best mutation rate for this definition of mutation is 2-4%.

5- NLP formulation

Lewin et al. [13], [14] have used an NLP formulation for maximum energy recovery (MER) and the present formulation is based on their method. Although this approach is efficient, they have not considered a search for minimum approach temperature in the network. Thus, in this work a search was added to find the optimum ΔT_{min}. In this method area calculations are not considered explicitly in the formulation and a penalty term was added to reduce costs as much as possible. In fact, this term modifies the objective function and relaxes some exchangers from pinching at ΔT_{min}. This penalty is a very important part of the formulation. The objective function is:

$$\text{Maximize} \quad \sum_{i=1}^{\text{no. of exch.}} X_i + \left(\sum_{i=1}^{2(\text{no. of exch.})} \Delta T \right) / \text{S.F.} \qquad (1)$$

Where X_i is the load of exchangers and ΔT is the approach temperature in the hot or cold ends of exchangers. S.F. is a scaling factor and must be large enough to ensure that the penalty term does not affect the main objective which is maximum energy recovery.

Constraints of this NLP are:

a) Energy balance for each exchanger on hot and cold streams. (Nonlinear if splitting occurs)

b) Energy balance for hot and cold utilities. Heaters and coolers are included in the formulation and if they are not needed, the optimization sets their loads to zero. (Linear)

c) Mass balance for splitters. (Linear)

d) Monotonic decrease or increase of temperatures on streams. (Linear)

e) Hot and cold end approach temperatures must be equal or greater than ΔT_{min} in each exchanger including utility exchangers. (Linear)

f) Energy balance at mixing points. (Linear)

For example, the constraints for Fig. 3 are:

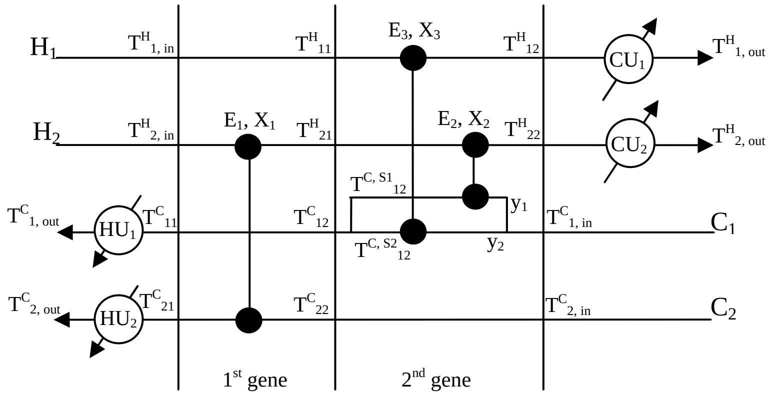

Figure 3. A network with three exchangers

a) Exchangers energy balance:

$$E_1: T^H_{2,\,in} - X_1/F^H_2 = T^H_{21} \quad , \quad T^C_{22} + X_1/F^C_2 = T^C_{21}$$

$$E_2: T^H_{21} - X_2/F^H_2 = T^H_{22} \quad , \quad T^C_{1,\,in} + X_2/(y_1F^C_1) = T^{C,\,S1}_{12}$$

$$E_3: T^H_{11} - X_3/F^H_1 = T^H_{12} \quad , \quad T^C_{1,\,in} + X_3/(y_2F^C_1) = T^{C,\,S2}_{12}$$

b) Utilities energy balance:

$$H_1: T^H_{12} - CU_1/F^H_1 = T^H_{1,\,out} \quad , \quad H_2: T^H_{22} - CU_2/F^H_2 = T^H_{2,\,out}$$

$$C_1: T^C_{11} + HU_1/F^C_1 = T^C_{1,\,out} \quad , \quad C_2: T^C_{22} + HU_2/F^C_2 = T^C_{2,\,out}$$

c) Mass balance on splitters:

$$y_1 + y_2 = 1$$

d) Monotonic decrease or increase in temperatures:

H_1: $T^H_{1,\,in} > T^H_{11} > T^H_{12} > T^H_{1,\,out}$

For other streams the same relations must written.

e) Minimum approach temperatures:

E_1: $T^H_{2,\,in} - T^C_{21} \geq \Delta T_{min}$, $T^H_{21} - T^C_{22} \geq \Delta T_{min}$

The same inequalities exist for E_2, E_3, CU_1, CU_2, HU_1 and HU_2

f) Mixing point energy balance:

$T^C_{12} = y_1 T^{C,\,s1}_{12} + y_2 T^{C,\,s2}_{12}$

In the above equations T_{ij} shows the temperature of i^{th} stream that exists at j^{th} gene, and F is heat capacity flow rate. According to the above, the overall algorithm is shown in Fig. 4.

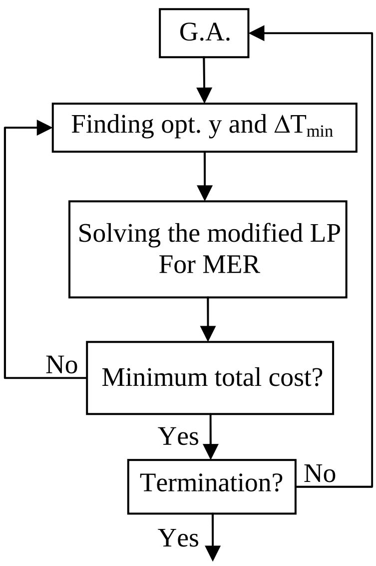

Figure 4. The algorithm for HEN synthesis

In Fig. 4, the G.A. produces different networks in each population and then each network is optimized by finding the related minimum annual cost which is the summation of area and utility costs. In this algorithm split ratios and minimum approach temperatures are not optimized simultaneously with exchanger heat loads. Instead an inner loop is utilized to find the best y and ΔT_{min}. In this loop the problem is converted to modified linear programming for finding MER. Therefore the NLP is converted to a search for y and ΔT_{min} and an LP for MER because the nonlinear terms in constraint (a) are eliminated by known split ratios.

Care must be taken for small split ratios because they import ill conditioning to the LP. So split ratios are bounded from 0.1 to 0.9 and a post analysis is needed for removing by-passes if they impose additional area to the network. Also the search range for ΔT_{min} is set to [0.1, 30]. At last, the fitness of each network is determined due to the best cost found by the inner loop.

In summary, this work has the following differences with the Lewin et al. method [13,14]:

a) Representation of network

b) Definition of mutation operator

c) Modification in LP by introducing a penalty term

d) Search for ΔT_{min}

e) Elimination of by-passes

These modifications help the algorithm to find better and more promising solutions and converge in relatively few iterations.

It is notable that this method is not based on pressure drop aspects and the constant heat transfer coefficients are assumed for the synthesis. In the rest of this paper some case studies are presented.

6- Case studies

Four case studies are solved by MATLAB codes. The first is an example solved by Yee and Grossmann [5] and Shirvakumar and Narasimhan [8]. The second and third belong to Lin and Miller [15] and the last is example 4S1 from Shenoy [3]. For all these problems counter current heat exchangers were considered. The mutation and crossover rates are set to 3 and 60% respectively and the number of chromosomes in each population is 20-30. To control the size of the problem only two branches are allowed in splitters.

6-1- Case 1

This case has two hot streams and two cold streams with power cost function and originally analyzed by Yee and Grossmann [5]. The data for this example are given in Table 1. Five genes are put into each chromosome for the synthesis of this network.

Table 1. Data for case 1

Streams	T^{in} (K)	T^{out} (K)	FC_p (kW/K)	Cost ($/kW/yr)
H_1	443	333	30	
H_2	423	303	15	
C_1	293	408	20	
C_2	353	413	40	
Steam	450	450		80
Water	293	313		20
U=.8 kW/m^2K for all exch. except ones involving steam				
U=1.2 kW/m^2K for matches involving steam				
Annualized cost ($/year) = $1000A^{0.6}$ for all exch. except heaters, A in m^2				
Annualized cost ($/year) = $1200A^{0.6}$ for heaters, A in m^2				
Chen's approximation is used for LMTD				

The final network has six exchangers and is shown in Fig. 5, in which the value of y_1 and ΔT_{min} are 0.33 and 4.7 K respectively. In this figure the underlined numbers are heat loads. The total cost of this network is 80250 $/yr, while Yee and Grossmann [5] reported a value of 80274 $/yr and Shirvakumar and Narasimhan's solution gives a value of 80851 $/yr [8].

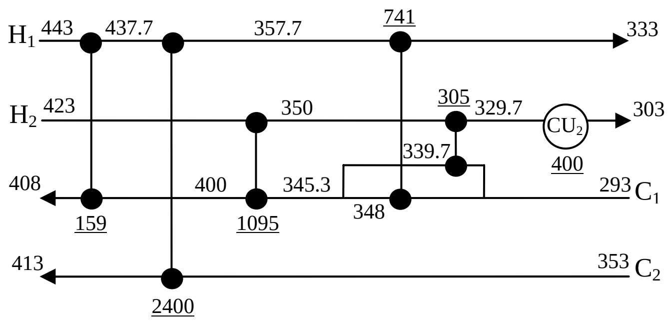

Figure 5. Optimal design for case 1

One of the advantages of the G.A. is that it does not produce only one solution. The second best network has six exchangers and one by-pass with a total cost of 81987 $/yr. By removing this by-pass the cost decreases to 81631 $/yr.

As shown in Fig. 6 the best solution is achieved after 43 iterations. Also, the average cost of populations vs. iterations is plotted in Fig. 7.

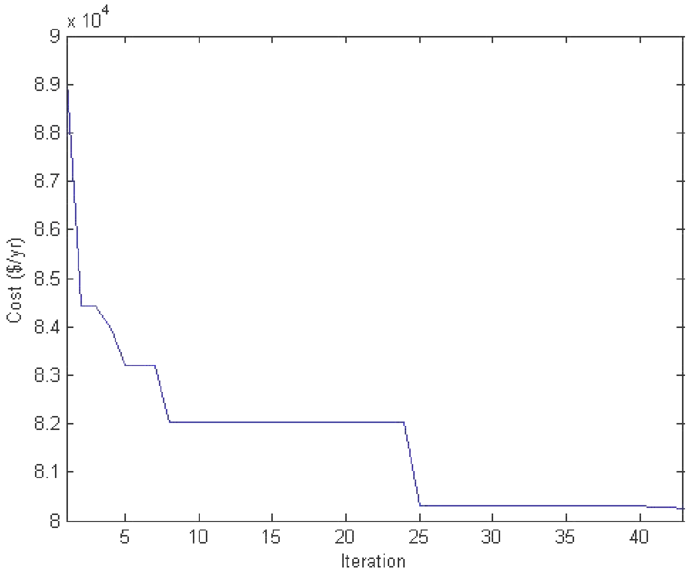

Figure 6. Minimum Cost vs. Iteration for case 1

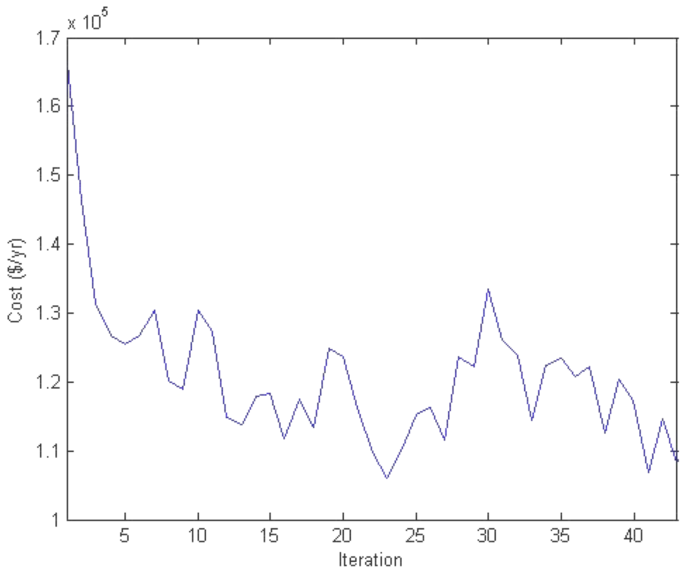

Figure 7. Average cost of populations vs. Iteration for case 1

The same solution may be achieved by the superstructures of Yee and Grossmann [5] if the number of stages is set to more than Max {NH, NC}, which is two in this problem.

6-2- Case 2

This example has been studied by Lin and Miller [15] with the optimal solution of 154997 $/yr and six exchangers. Table 2 includes stream and cost data for this case and Fig. 8 shows the best solution found by this coupled method in which each chromosome has five genes.

Table 2. Data for case 2

Streams	T^{in} (K)	T^{out} (K)	FC_p (kW/K)	Cost ($/kW/yr)
H_1	650	370	10	
H_2	590	370	20	
C_1	410	650	15	
C_2	350	500	40	
Steam	680	680		80
Water	300	320		15
U=0.5 kW/m^{2}K for all exch. except ones involving steam				
U=0.833 kW/m^{2}K for matches involving steam				
Annualized cost ($/year) = 5500+150A for all exch., A in m^2				
Chen's approximation is used for LMTD				

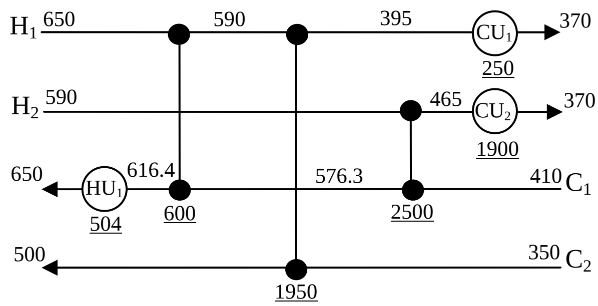

Figure 8. Best Network for case 2

The genetic algorithm finds the same structure as Lin and Miller [15] with the annual cost of 156730 $/yr. As the NLP formulation do not optimize the area of exchangers simultaneously with other parameters, an increase of 1.1% occurs in total annual cost.

6-3- Case 3

This case is solved by Lin and Miller [15] and Zamora and Grossmann [19] with the global solution of 82043 $/yr and five exchangers. The problem has three hot streams and two cold streams. Also, logarithmic mean temperature difference is replaced with an arithmetic mean and each stream has a specific heat transfer coefficient. Table 3 includes information from streams and cost data.

The first solution created by the G.A. has five exchangers which are indicated in Fig. 9. The cost of this network is 82151 \$/yr and an error of 0.13% occurs due to the NLP formulation. If the penalty term is not used in the objective function the cost of the best network increases about 16%, showing the important role of this term in the formulation.

Table 3. Data for case 3

Streams	**T^{in} (K)**	**T^{out} (K)**	**h (kW/(m^2°C))**	**FC_p (kW/K)**	**Cost (\$/kW/yr)**
H_1	159	77	0.1	2.285	
H_2	267	80	0.04	0.204	
H_3	343	90	0.5	0.538	
C_1	26	127	0.01	0.933	
C_2	118	265	0.5	1.961	
Steam	300	300	0.05		110
Water	20	60	0.2		10
Annualized cost (\$/year) = 7400+80A for all exch., A in m^2					

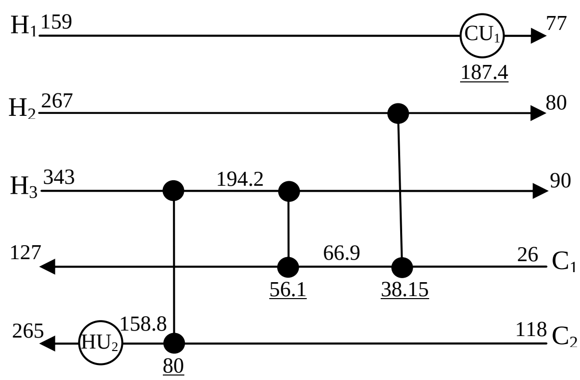

Figure 9. Best network for case 3

6-4- Case 4

As the number of splitters increases in a network, difficulties arises due to some industrial aspects like the control of temperatures in mixers and piping of the network. This case is solved by Shenoy [3] and his network has six exchangers and two splitters with an annual cost of 240030 \$/yr. In this example, each gene is forced to have only one splitter to reduce the total number of splitters in the network. Table 4 shows the data of this problem.

Fig. 10 indicates the best solution with 5 exchangers and no splitters. The cost of this network is 241923 $/yr, which is 0.8% greater than the solution of Shenoy [3]. The second best network achieved by this method has six exchangers and one splitter with a cost of 245140 $/yr. As can be seen from the first and second solution, the restriction in the number of splitters in each gene does not import considerable increase in the annual cost. So it is useful to reduce the number of splitters to avoid difficulties in control or piping aspects.

Table 4. Data for case 4

Streams	T^{in} (°C)	T^{out} (°C)	FC_p (kW/°C)	Cost ($/kW/yr)
H_1	175	45	10	
H_2	125	65	40	
C_1	20	155	20	
C_2	40	112	15	
Steam	180	179		120
Water	15	25		10
U=0.1 kW/m^{2}K for all exchangers				
Cost ($) = 30000+750A$^{0.81}$ for all exchangers, A in m^2				
Plant life time: 5 yr, Rate of interest: 10%				
LMTD is used for area calculations				

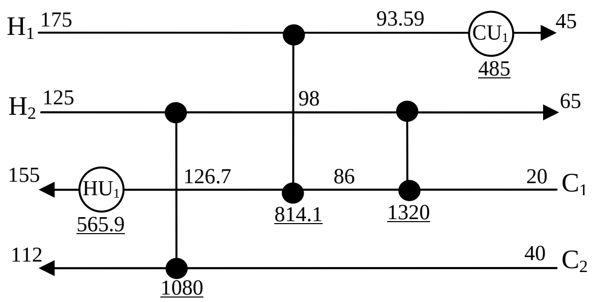

Figure 10. Best solution find for case 4 with restriction in no. of splitters

7- Conclusion

In this paper a new coupled G.A.-NLP formulation is presented in which new representation for HEN is considered by definition of exchangers as genes. The NLP is an optimization problem for the maximization of energy recovery with a penalty term that relaxes exchangers from pinching at ΔT_{min}. Although G.A. produces good solutions, there is no guarantee to find the optimum solution and it is unnecessary to formulate the NLP so restrictively. So in this work areas were not optimized simultaneously with other parameters. This kind of formulation greatly reduces the complexity of the problem.

The case studies show that this method is very efficient and has a maximum increase of less than 2% in HEN annual cost. One of the advantages of this method is that it does not require an initial guess for structural and continuous variables because of the use of the G.A. and the simplex method for the optimization of continuous parameters. Many solvers have problems in the convergence of binary and continuous variables, and their robustness weakens when the size of networks increases. But this work is not sensible to the size of problems and is very robust because the usual NLP is replaced with an LP which is easier to optimize. So this method can be extended to industrial problems in future works.

Nomenclature

C_i	i^{th} cold stream, $i=1\ldots,n_C$
CU_i	heat load of i^{th} cold utility exchanger, $i=1\ldots,n_H$
E_i	i^{th} exchanger
F	heat capacity flow rate
H_i	i^{th} hot stream, $i=1\ldots,n_H$
HU_i	heat load of i^{th} hot utility exchanger, $i=1\ldots,n_C$
LMTD	logarithmic mean temperature difference
N_C	number of cold streams
N_H	number hot streams
T_{ij}	temperature of i^{th} stream that exits at j^{th} gene
U	overall heat transfer coefficient
X_i	heat load of i^{th} exchanger
y	split ratio
ΔT	approach temperature in hot or cold end of an exchanger
ΔT_{min}	minimum approach temperature in a network

Indices

C	cold
H	hot
in	inlet
out	outlet
s_i	i^{th} branch of a splitter

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