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Simulation of Water Coning in Oil Reservoirs Using a Corrected IMPES Method

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Abstract

Implicit pressure-explicit saturation method (IMPES) is widely used in oil reservoir simulation to study the multiphase flow in porous media. This method has no complexity compared to the fully implicit method, although both of them are based on the finite difference technique. Water coning is one the most important phenomenon that affects the oil production from oil reservoirs having a water drive source. Since the water coning affects final oil recovery, identification of this phenomenon is very important. In order to study this phenomenon, one should determine the critical production rate, the breakthrough time and watercut percentage. The scale of the problem hinders the numerical simulations, IMPES included, for a long running time. A corrected IMPES method is used here to overcome the long running time problem by choosing larger the time step for the coning problem. A water-oil phase flow system in the cylindrical coordinate that is commonly used to simulate water coning phenomenon is solved by the corrected IMPES method. The validity of the model is checked against Aziz and Settari's model, which is based on a complicated fully implicit method. The effects of the production rate and the thickness of the oil zone on the breakthrough time have been investigated. The results were found to be in good agreement with the results of previous studies.

Keywords: *Water Coning, Simulation, Oil Reservoirs, IMPES*

Introduction

Water encroachment into reservoirs and the simultaneous production of oil and water is one of the major problems in reservoir engineering. The appearance of water in oil formation due to its presence in aquifiers is considered as water coning, (this is due to its cone-Shape) and is widely found in oil reservoirs. In the study of coning, three things should be determined. First, the maximum oil production rate at which a well can be produced without coning any water. This is called "Critical Rate". Second, if the well produces above the critical rate, the breakthrough time, and third, the watercut performance after breakthrough [1].

Muskat and Wyckoff [2], Arthur [3], Chaney et al. [4] and Chierici and Ciucci [5] used graphical solutions for critical rate determinations, while Meyer and Garder [6]

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and Schols [7] calculated the critical rate using theoretical solution. The prediction of watercut performance is usually complicated and costly. Sobocinski and Cornelius [8], and Bournazel and Jeanson [9] proposed empirical correlations for watercut performance predictions. Letkeman et al. [10] used a numerical coning model to match coning history and to investigate various completion and production techniques. Miller and Roger [11] used a numerical simulator to study the effect of different reservoir parameters on coning performance.

Simulation of water coning is achieved by solving water-oil flow partial differential equations in a radial system using the finite difference technique [12]. Since the coning phenomenon occurs in the vicinity of the wellbore which leads to dramatic variation of pressure and saturation, to increase the accuracy of the calculated results the selected grid size must be very small in that region. Besides, the high flow velocity in the vicinity of the wellbore is very large compared to the rest of the reservoir, therefore the results are very sensitive to the selection of the numerical method to skip the numerical instability. It has been shown that the computational instability mostly occurs in the finite difference method when the saturation-dependent parameters are set constant during a time step. [12]. Therefore, the explicit method to solve these equations is not suggested because of the instability problem. Welge and Weber [13] applied an arbitrary limitation on the maximum saturation change over a time step to solve the water coning problem using the explicit method. While this method could be used for certain types of problems such as one-dimensional water-flooding, it is not rigorous and is not generally applicable.

The implicit methods are mathematically complex and time consuming techniques. Blair and Weinaug [14] explored the use of explicitly determined coefficients and formulated a coning model using implicit mobilities and further utilizing the Newtonian iteration method to calculate the pressure and saturation history. While this method is rigorous, achieving the convergence criteria on certain problems is difficult and requires very restricted conditions on time-step size selection. It is therefore of practical interest to explore a method of eliminating the instability found in the IMPES method by utilizing the production and transmissibility terms calculated in the new time step using Taylor' series extension. It is intended here to correct the production terms alone to find its effect on the permissible time step compared to the original IMPES method.

Mathematical Model

The mathematical model used for the coning studies is two phase flow partial differential equations that are obtained by combination of continuity and extended Darcy's law for each phase. In the cylindrical coordinates the equations are: [15, 16]

$$\frac{\partial}{\partial z}\left(K_v \frac{K_{ro}}{\mu_o B_o}\left(\frac{\partial P_o}{\partial z} - \rho_o g\right)\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r K_h \frac{K_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial r}\right) - q_o = \frac{\partial}{\partial t}\left(\varphi \frac{s_o}{B_o}\right) \quad (1\text{-a})$$

$$\frac{\partial}{\partial z}\left(K_v \frac{K_{rw}}{\mu_w B_w}\left(\frac{\partial P_w}{\partial z} - \rho_w g\right)\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(r K_h \frac{K_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial r}\right) - q_w = \frac{\partial}{\partial t}\left(\varphi \frac{s_w}{B_w}\right) \quad (1\text{-b})$$

Additional relations are required to solve Equations (1a) and (1b) such as:

$$P_c = f(s_w) = P_o - P_w \quad (2)$$

$$s_w + s_o = 1 \quad (3)$$

To solve non-linear Equations (1a) and (1b), they are changed to linear form using the finite difference technique and are then solved by the IMPES numerical method. Rewriting Equations (1a) and (1b) in discrete form:

$$\left[\Delta T_o^n \Delta\varphi_o^{n+1}\right]_{ik} = \frac{V_{p\,ik}}{\Delta t}\Delta_t\left(\frac{S_o}{B_o}\right)_{ik}^n + q_{oik} \tag{4}$$

$$\left[\Delta T_w^n \Delta\varphi_w^{n+1}\right]_{ik} = \frac{V_{p\,ik}}{\Delta t}\Delta_t\left(\frac{S_w}{B_w}\right)_{ik}^n + q_{wik} \tag{5}$$

For l^{th} phase:

$$\Delta T_l \Delta\varphi_l = \Delta_r T_{rl}\Delta_r\varphi_l + \Delta_z T_{zl}\Delta_z\varphi_l \tag{6}$$

$$\varphi_l = P_l - \rho_l g z \tag{7}$$

$$\left(\Delta_r T_{rl}\Delta_r\varphi_l\right)_{ik} = \left[T_{rl\,i+\frac{1}{2}}\left(P_{l\,i+1} - P_{l\,i}\right) + T_{rl\,i-\frac{1}{2}}\left(P_{l\,i-1} - P_{l\,i}\right)\right]_k \tag{8}$$

$$T_{rl\,i+\frac{1}{2},k} = 2\pi\pi_{h\ i+\frac{1}{2},k}\frac{(z_{k+\frac{1}{2}} - z_{k-\frac{1}{2}})}{\ln\frac{r_{i+1}}{r_i}}\left[\frac{k_{rl}}{\mu_l B_l}\right]_{i+\frac{1}{2},k} \tag{9-a}$$

$$T_{rl\,i-\frac{1}{2},k} = 2\pi\,k_{h\ i-\frac{1}{2},k}\frac{(z_{k+\frac{1}{2}} - z_{k-\frac{1}{2}})}{\ln\frac{r_i}{r_{i-1}}}\left[\frac{k_{rl}}{\mu_l B_l}\right]_{i-\frac{1}{2},k} \tag{9-b}$$

$$\begin{aligned}(\Delta_z T_{zl}\Delta_z\varphi_l)_{ik} = {} & [T_{zl\,k+\frac{1}{2}}(P_{l\,k+1} - P_{l\,k} - \gamma_l(z_{k+1} - z_k)) \\ & + T_{zl\,k-\frac{1}{2}}(P_{l\,k-1} - P_{l\,k} - \gamma_l(z_{k-1} - z_k))]_i\end{aligned} \tag{10}$$

$$T_{zl\,i,k+\frac{1}{2}} = \pi k_{v\,i,k+\frac{1}{2}}\frac{(r^2_{i+\frac{1}{2}} - r^2_{i-\frac{1}{2}})}{(z_{k+1} - z_k)}\left[\frac{k_{rl}}{\mu_l B_l}\right]_{i,k+\frac{1}{2}} \tag{11-a}$$

$$T_{zl_{i,k-\frac{1}{2}}} = \pi k_{v\,i,k-\frac{1}{2}} \frac{(r^2_{i+\frac{1}{2}} - r^2_{i-\frac{1}{2}})}{(z_k - z_{k-1})} \left[\frac{k_{rl}}{\mu_l B_l}\right]_{i,k-\frac{1}{2}} \tag{11-b}$$

$$r_{i+\frac{1}{2}} = \left[r^2_{i+1} - r_i^2 \right]^{\frac{1}{2}} \tag{12}$$

$$z_{k+\frac{1}{2}} = \frac{1}{2}(z_k + z_{k+1}) \tag{13}$$

$$V_{P_{ik}} = \varphi_{ik}\pi(r^2_{i+\frac{1}{2}} - r^2_{i-\frac{1}{2}})(z_{k+\frac{1}{2}} - z_{k-\frac{1}{2}}) \tag{14}$$

$$\Delta_t\left(\frac{S_w}{B_w}\right)^n_{ik} = S^n_w\left(\frac{1}{B_w}\right)'(P^{n+1}_w - P^n_w) - \frac{S'_w}{B^{n+1}_w}(P^{n+1}_o - P^{n+1}_w - P^n_o + P^n_w)) \tag{15-a}$$

$$\Delta_t\left(\frac{S_o}{B_o}\right)^n_{ik} = (1 - S^n_w)\left(\frac{1}{B_o}\right)'(P^{n+1}_o - P^n_o) - \frac{S'_w}{B^{n+1}_o}(P^{n+1}_o - P^{n+1}_w - P^n_o + P^n_w) \tag{15-b}$$

Where $S'_w = \frac{dS_w}{dP_c}$ and $\left(\frac{1}{B_l}\right)' = \frac{d(1/B_l)}{dP_l}$.

Combining the Equations (4-5), an implicit equation is obtained based on the pressure of water phase. Solving this Equation and using Equation (5), the saturation is calculated explicitly.

Based on the Blair and Weinaug's work [14], explicit handling of parameters dependent on the saturation in the IMPES method would cause instability. This could be prevented if the corrected type of IMPES is used. In the corrected form of the IMPES method the production terms are calculated for the new time step (n+1) and combined with Equation (5) to find the amount of saturation for each grid block.

Water and oil production terms in Equations (4-5) are employed here in the form of the total production and fractional flow terms as shown below:

$$q = q_w + q_o \tag{16}$$

$$f_w = \frac{\frac{k_{rw}}{\mu_w}}{\frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o}} \tag{17}$$

In the first step, the fractional flow term in the new time step (n+1) is estimated using the Taylor's series extension applied to the old value of fractional flow term (n).

$$f_{w\,n+1} = f_{w\,n} + f'(s_{w\,n+1} - s_{w\,n}) \qquad (18)$$

$$f' \approx \frac{f_{w\,n+1} - f_{w\,n}}{s_{w\,n+1} - s_{w\,n}} \qquad (19)$$

Where, f ' is the slope of fractional flow curve at $S_{w\,n}$.

In the next step, having $f_{w\,n+1}$, the $q_{w\,n+1}$ is estimated using the following equation:

$$q_{w\,n+1} = q_{w\,n} + qf'(s_{w\,n+1} - s_{w\,n}) \qquad (20)$$

This eventually leads to the calculation of saturation at new time step for each block by inserting the $q_{w\,n+1}$ in Equation (5).

$$S_{wik}^{n+1} = S_{wik}^{n} + \frac{1}{(q\,f' + Csww_{ik})}$$

$$\left[\left[\Delta T_w \Delta\varphi_w^{n+1}\right]_{ik} - Cpow_{ik}(Po_{ik}^{n+1} - Po_{ik}^{n}) - q_{wik}\right] \qquad (21)$$

$$Cpow_{ik} = \frac{V_{p\,ik} Sw_{ik}}{\Delta t}\left[\frac{d(1/B_w)}{dP_w}\right]_{ik} \qquad (22)$$

$$Csww_{ik} = \frac{V_{p\,ik}}{B_{wik}\Delta t} - \left(\frac{dP_{cow}}{dSw}\right)_{ik} Cpow_{ik} \qquad (23)$$

Case Study

A coning calculation is performed using the data obtained by Blair and Weinaug which is known as the standard data used by many researchers [16,17,18].The calculated results from the proposed model in this work are then compared with Aziz and Settari's model (Appendix A) .The water-oil coning data are given in Tables 1 and 2 and Figures 1 and 2. [18]

Table 1. Water-Oil coning problem data [18]

Oil density	51.54 (lb_m / ft^3)
Water density	62.4 (lb_m / ft^3)
Oil Compressibility Coefficient	$1\times10^{-5}\ psi^{-1}$
Water Compressibility Coefficient	$3\times10^{-6}\ psi^{-1}$
Oil viscosity	$0.31cp$
Water viscosity	$0.34cp$
porosity	0.207
Well radius	$2.45\,ft$
Drainage radius	$1300\,ft$
Initial reservoir pressure	$2000\,psia$
Reservoir thickness	$365\,ft$
Oil zone thickness	$160\,ft$
Vertical permeability	$100md$
Horizontal permeability in oil zone	$1000md$
Horizontal permeability in water zone	$5000md$
Production flow rate	$6000RB / day$

Table 2. Saturation functions table [18]

Sw	Krw	Kro	Pc
0.15	0	0.95	1.2
0.2	0.004	0.75	0.66
0.25	0.0102	0.5875	0.54
0.3	0.0166	0.4462	0.47
0.35	0.0232	0.3325	0.42
0.4	0.0305	0.245	0.38
0.45	0.0392	0.177	0.34
0.5	0.0497	0.12	0.3
0.55	0.063	0.0724	0.27
0.6	0.0797	0.03745	0.24
0.65	0.1	0.01627	0.205
0.7	0.1244	0.00564	0.17
0.75	0.1525	0.00077	0.12
0.775	0.1698	0.00038	0.08
0.788	0.1784	0.00019	0
0.8	0.187	0	-0.2

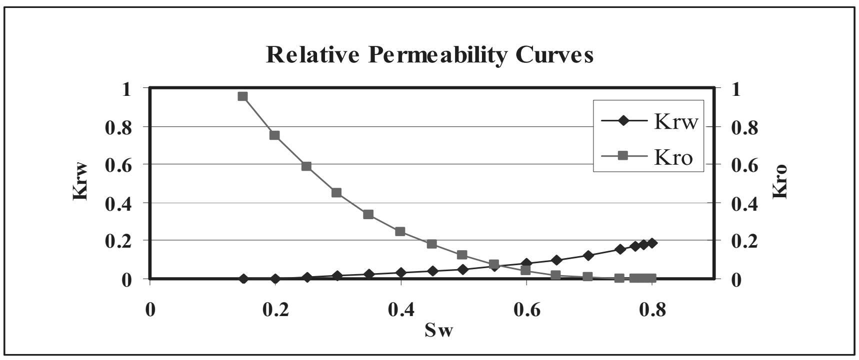

Figure 1. Relative Permeability Curves [18]

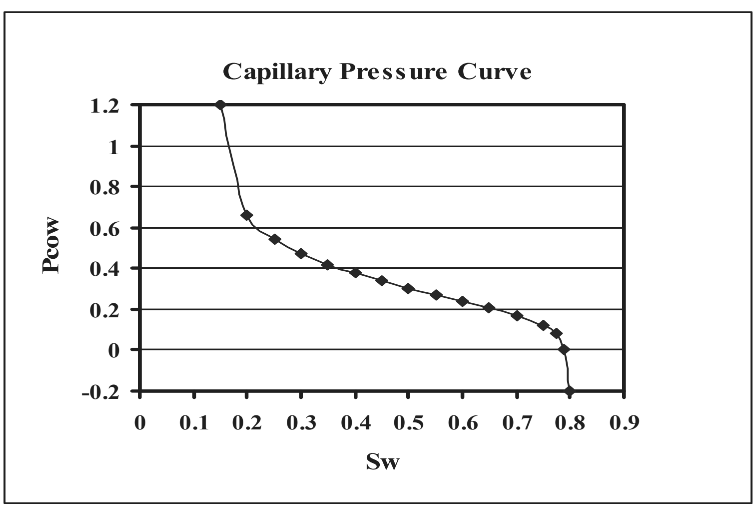

Figure 2. Capillary Pressure Curve [18]

The calculated results are shown in Figs. 3-4. Figure 3 shows the water saturation history in the production grid block that reveals the water breakthrough time. This clearly indicates that the water breakthrough occurs as the water saturation exceeds the critical saturation. Figure 4 displays the water-oil ratio (WOR) as a function of time. The breakthrough time is also shown in Fig. 4 which is the start of the water production and deviation of WOR from zero value. The results found by Aziz and Settari were also shown in Figs. 3-4 which reveal very good agreement with the proposed model.

Sensitivity analysis was done on the calculated results to evaluate the effects of oil thickness and the production flow rate. The results are shown in Fig. 5, revealing the crucial effects of these two parameters. As the oil thickness increases, so does the breakthrough time, and when production rate increases the breakthrough time decreases.

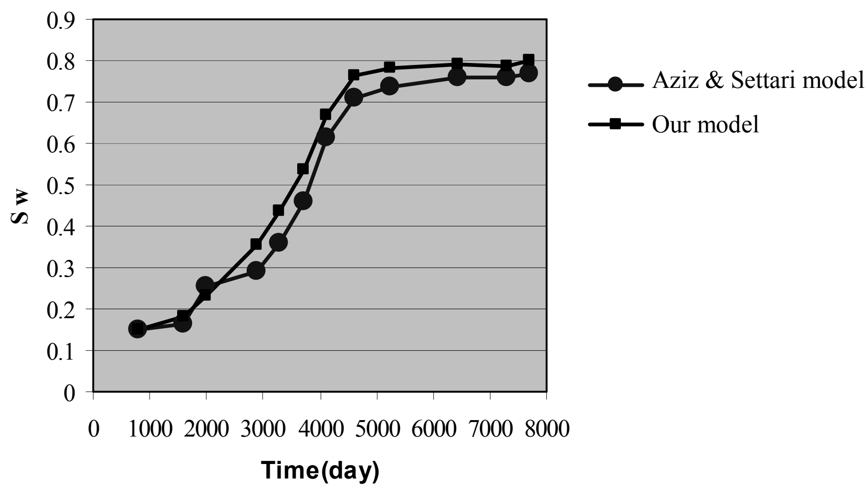

Figure 3. Water saturation versus time in grid block connected to wellbore

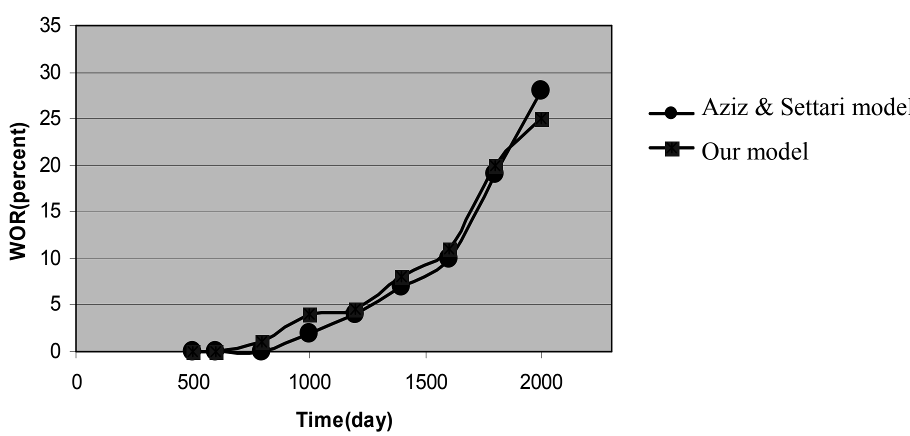

Figure 4. WOR versus Time

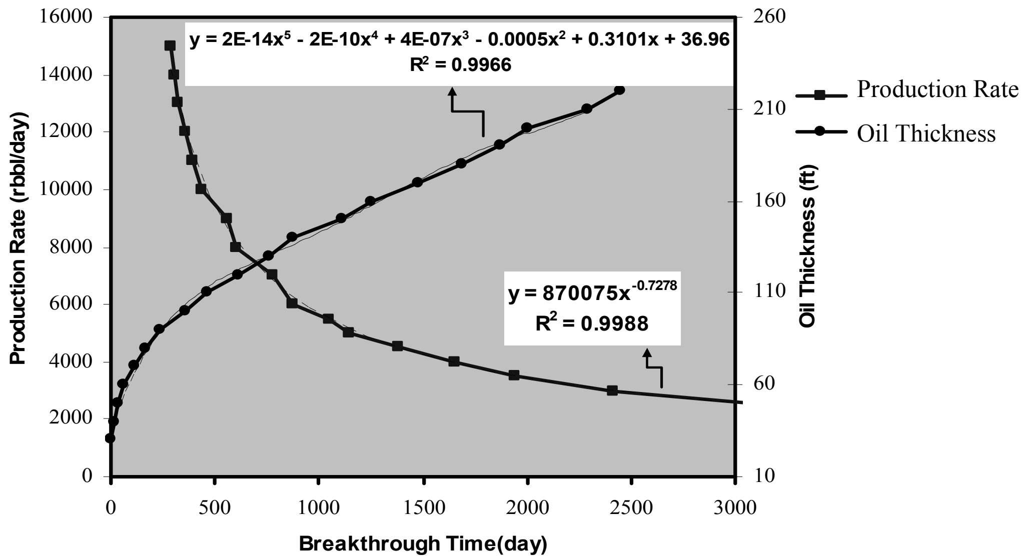

Figure 5. Oil Thickness and Production Rate versus Breakthrough Time

Conclusions

IMPES applied on partial differential equations for oil-water flow in a cylindrical coordinate was corrected by using a new procedure to calculate the production term in a new time level(n+1). This procedure enhanced the time step size up to approximately five times that of previous works. Figures 3-4 confirm the accuracy of the results as they are matched with the well-known results found by Settari and Aziz. Besides, the physical concept of the coning phenomenon could be seen clearly in the sensitivity analysis done on the breakthrough time as a function of oil thickness and production rate. As the distance between the production grid block and water-oil contact is increased, the breakthrough time will be postponed, and also, when the production rate is decreased, the breakthrough time will be delayed.

Nomenclature

i	*spatial position in r direction*
k	*spatial position in z direction*
l	*o,w*
o	Oil
w	Water
K_v	Vertical permeability ($0.001127 \times$ md)
K_h	Horizontal permeability ($0.001127 \times$ md)
K_r	Relative permeability
P	pressure ($psia$)
B	Formation volume factor (RB / STB)
ρ	density ($lb / cuft$)
q	source/sink terms (STB / day)-(positive for water and negative for oil)
s	Saturation
φ	Porosity
n	Old time
$n+1$	New time
Δ	Differential Operator
$V_{p\,ik}$	Pore volume of block (i, k) ($ft^{۳}$)

Appendix A

Brief description of Aziz and Settari's Model:

In order to simulate the water coning simulation, a fully implicit treatment of transmissibilities was employed while solving the cylindrical flow system of the method, the fully implicit method utilizes the pressure and saturation implicitly in new time(n+1). Therefore, more running time is required and mathematical complexity is greater.

$$\left[\Delta T_o^{n+1}\Delta\varphi_o^{n+1}\right]_{ik} = \frac{V_{p\,ik}}{\Delta t}\Delta_t\left(\frac{S_o}{B_o}\right)_{ik}^{n} + q_{oik}$$

$$\left[\Delta T_w^{n+1}\Delta\varphi_w^{n+1}\right]_{ik} = \frac{V_{p\,ik}}{\Delta t}\Delta_t\left(\frac{S_w}{B_w}\right)_{ik}^{n} + q_{wik}$$

These equations have been developed in residual format and the iterative method or approximate direct method were used in an attempt to solve them. Moreover, two factors have been considered: (1) the outlet effect, which requires that capillary pressure to approach zero at the sand face, and (2) the compatibility condition, as the vertical pressure gradeint in the well must be the same as the pressure gradient at the reservoir/wellbore boundary.

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