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Development of an explicit Eulerian method for treating particle-laden turbulent flow in dust collectors

M. R. Talaie
Isfahan University, School of Engineering, Chemical Engineering Department Isfahan, Iran.

Abstract

Prediction of particle removal efficiency in all kinds of dust collectors includes mathematical modeling of particle-laden flow. One of the main problems for such modeling is to find particle velocity distribution which can be predicted using a particle momentum balance equation. A common simple alternative is to use equilibrium or settling velocities. In this work, a comprehensive mathematical model for particle removal in double-stage electrostatic precipitators was developed. Analysing the results of this model shows that the equilibrium assumption can be considered as a reliable method for a Stokes number less than 0.001. Also, for a higher Stokes number, an explicit Eulerian method was developed for the evaluation of particle velocities explicitly. This method eliminates solving particle momentum balance equations which is a relatively time-consuming step in mathematical modeling of dust collectors.

Keywords: *Mathematical modeling, Dust collectors, Particle-laden flow, Particle velocity field, Eulerian approach*

Introduction

Particle-laden flow can be observed in many practical engineering problems. A comprehensive mathematical model of all kinds of dust collectors such as a cyclone, settling chamber and electrostatic precipitator involve mathematical modeling of particle-laden flow. There is a spectrum of methods that can be used to predict particle dispersion through a fluid. The choice of method depends on the size of the particle compared to the characteristic time-scale of the surrounding flow. Ferry and Balachandar [1] introduced a classification of these methods. Fluid and particle flows may have interaction depending on the relative size and concentration of particles. However, for the case of dust collectors, regarding practical particle loading, there is no significant interaction according to Squires and Eaton [2]. Eulerian approach is one of the main methods in treating particle-laden flow and is based on the continuum model. It has some advantages and deserves to be used as a powerful tool to model particle dispersion. The evaluation of the particle velocity field is one of the main requirements to use this method. The particle velocity field depends on particle size and the magnitude of the external force exerted on particles. If the external force is too low or particles are very small they can be considered to move with the fluid and be spatially well mixed. In this case the particle velocity field is identical to the fluid velocity field (e. g. capture of fine particulate in impingement filters). For larger particles and a higher exerted force, the particle velocity can no longer be considered equal to fluid velocity. Nonetheless, the particles are still well represented by a con-tinuum model. The particle velocity field can be evaluated based on the assumption that particles reach their equilibrium velocity instantaneously. This assumption is valid exactly when particle size reduces to mole-cule size, and can deviate from real situation as the size of particle and external force increase. Ferry and Balach-andar [1] called this approach fast Eulerian method, because particle velocities can be calculated explicitly. If the equilibrium as-sumption is not valid, the particle mo-mentum balance equation should be solved to find the particle velocity field. Maxey [3] introduced an expansion of particle mo-mentum equation in terms of fluid velocity and particle time-scale. It can be used to explicitly calculate particle velocity field. He used a first order approximation of this expansion to show the accumulation of dense particles in region of low vorticity and high strain [3]. Druzhinin [4] used a second order approximation. Ferry and Balachandar [1] modified the expansion to include other effects, such as added mass, Basset history and Saffman lift forces. Ferry and Balach-andar [5] developed an expansion for particle velocity in terms of the fluid velocity with dimensionless particle time-scale. They sho-wed the rapid convergence obtained by this method compared to that obtained by the Lagrangian method to show its capability.

The main objectives of this study is to find the condition under which equilibrium assump-tion is valid and to develop a new fast Eulerian method which can be used to evaluate particle velocities explicitly. Using the fast Eulerian method eliminates solving particle momentum balance equation, which can be time-consuming for the numerical simulation of dust collector performance.

In order to show the applicability of this method, numerical simulation of a double-stage electrostatic precipitator was perfor-med. The results were compared by those obtained by solving the particle momentum equation.

Theoretical approach

Generally, in the Eulerian approach, the particle velocity field can be calculated using the following vector equation expressing the particle momentum balance with neglecting diffusion terms [4]:

$$U_P \cdot \nabla U_P = \frac{\frac{1}{2} C_{Df} \rho \frac{\pi}{4} D_P^2}{m_P} \left| U - U_P \right| (U - U_P) + \frac{f}{m_P} \tag{1}$$

In the above equation, U, Up and f are fluid velocity vector, particle velocity vector and external force vector respectively.

External forces accelerate particles until their velocities reach to settling or equilibrium velocities. Because at equilibrium $du_P/dt = 0$, the equilibrium velocity can be calculated from the the following equation expressing the Lagrangian momentum balance equation at the steady-state:

$$\frac{\frac{1}{2} C_{Df} \rho \frac{\pi}{4} D_P^2}{m_P} \left| U - U_{Ps} \right| (U - U_{Ps}) + \frac{f}{m_P} = 0 \tag{2}$$

Using Stokes law for the drag coefficient ($C_{Df} = 24/Re_p$), equilibrium velocity can be calculated by the following equation:

$$U_{Ps} = U + \frac{f}{3\pi D_P \mu} \tag{3}$$

For a Reynolds number greater than 0.1, Stokes law cannot be valid. The relations develped by Cliff and Gauvin [8] for the particle drag coefficient was used in this work. This relation is as follows:

$$\begin{cases} C_{Df} = \frac{24}{Re_P} & Re_P \langle 0.1 \\ C_{Df} = \frac{24}{Re_P}(1 + 0.15 Re_P^{0.687}) & 0.1 \langle Re_P \langle 1000 \end{cases} \quad (4)$$

The above equation has been used also by Suh and Kim [6] to simulate the performance of electrostatic precipitators. Using this relation the following equation for particle equilibrium velocity was obtained:

$$U_{Ps} = U + \frac{f\, g}{3\pi D_P \mu} \quad (5)$$

The value of g in the above equation can be found by the following relation:

$$\begin{cases} g = 1 & 0.1 \succ Re_P \\ g = \left[1 + Re_P^{0.687}\right] & 0.1 \prec Re_P \prec 1000 \end{cases} \quad (6)$$

The particle Reynolds number is given by the following equation:

$$Re_p = \frac{\rho \left| U - U_p \right| D_p}{\mu} \quad (7)$$

Introducing dimensionless variables $U_P^* = U_P/u_0$, $U^* = U/u_0$, $x^* = x/L$, $y^* = y/L$, $f^* = f/f_0$ and using Equation 4 for the drag coefficient, C_{Df}, into Equation 1 yields:

$$U_P^* \cdot \nabla U_P^* = \frac{g}{Stk}(U^* - U_P^*) + \frac{F^*}{Stk} f^* \quad (8)$$

Where Stk is the Stokes number which represents the ratio of inertia to drag force ($\frac{\rho_P C_C u_0 D_P^2}{18 \mu L}$), and F^* is a dimensionless number representing the ratio of exerted force to drag force (exerted force is gavity for settling chambers, centrifugal force for cyclones and electrostatic force for electrostatic precipitators). In electrostatic precipitators F^* is electrostatic number, Es ($\frac{C_C q_0 E_0}{3\pi \mu u_0 D_P}$), which has been introduced by Kihm et. al [7]. The above equation shows that for low Stk, equilibrium assumption can be valid and particle velocity can be calculated by Equation 5 explicitly. However, for higher values of Stk, Equation 8 should be solved to find particle velocity accurately.

Development of explicit Eulerian method

Subtracting Equation 1 from Equation 2 using Stokes law for the drag coefficient yields the new form of the momentum equation. This equation in dimensionless form is as follows:

$$U_P^* = U_{Ps}^* - Stk\, U_P^* \cdot \nabla U_P^* \quad (9)$$

Particle velocity can be calculated based on equilibrium velocity as follows:

$$U_P^* = U_{Ps}^* + w^* \quad (10)$$

where w is called the residual velocity vector, which is the difference between real particle velocity and equilibrium velocity.

By combining Equations 9 and 10 the following equation was obtained:

$$U_P^* = U_{Ps}^* - Stk\,(U_{Ps}^* \,.\, \nabla U_{Ps}^* + w^* \,.\, \nabla U_{Ps}^* + U_{Ps}^* \,.\, \nabla w^* + w^* \,.\, \nabla w^*) \qquad (11)$$

The first order approximation of the above equation can be obtained by eliminating the three last terms shown in paranthesis. The second approximation can be obtained by replacing the equation obtained by the first order approximation into Equation 9. However, these approximations cannot be used for the accurate prediction of the velocity field.

New explicit Eulerian method developed for high Stokes number

In order to modifiy this approximation the following relation was introduced:

$$U_P^* = U_{Ps}^* - c\ g\ Stk\left(U_{Ps}^* \cdot \nabla U_{Ps}^*\right) \qquad (12)$$

where c is a function of Stokes number. This function was obtained, tentatively, by minimizing the root mean square error of the particle velocity field for the case of a double-stage electrostatic precipitator:

$$c = (-1.33\,Stk\frac{|u_{pa}|}{u_0} + 1.2)\frac{|u_{pa}|}{u_0} \qquad (13)$$

where u_{pa} can be obtained by the following equation:

$$u_{pa} - 1.66\,Stk + 1.2 \qquad (14)$$

The value of g in Equation 12 can be attained by using Equation 6 and based on the following definition for Rep:

$$Re_P = \frac{|u_{Pa} - U|\ D_P}{\nu} \qquad (15)$$

Basic equations for numerical simulation of a double-stage ESP

The two components of vector equation 1 in two-dimensional form, and by considering diffusion terms, can be expressed by the following equations:

$$\frac{\partial\left[u_p\left(u_pC_p - E_p\frac{\partial C_p}{\partial x}\right)\right]}{\partial x} + \frac{\partial\left[u_p\left(v_pC_p - E_p\frac{\partial C_p}{\partial y}\right)\right]}{\partial y} = \frac{3}{4}\frac{C_{Df}\rho_fC_p}{D_p\rho_p}|U - U_p|(u - u_p) + \frac{Q_pE_xC_p}{m_p} \qquad (16)$$

$$\frac{\partial\left[v_p\left(u_pC_p - E_p\frac{\partial C_p}{\partial x}\right)\right]}{\partial x} + \frac{\partial\left[v_p\left(v_pC_p - E_p\frac{\partial C_p}{\partial y}\right)\right]}{\partial y} = \frac{3}{4}\frac{C_{Df}\rho_fC_p}{D_p\rho_p}|U - U_p|(v - v_p) + \frac{Q_pE_yC_p}{m_p} \qquad (17)$$

where up and vp are the x-component and y-component of the particle velocity vector, Up, and u and v are the x-component and y-component of fluid velocity vector, U.
In order to make the results of the above equations independent of particle concentration, Cp, the turbulent diffusion term of the above equations can be neglected with respect to the convective term. Neglecting the diffusion terms in the two above equations, yields:

$$u_p\left(\frac{\partial(C_p u_p)}{\partial x}+\frac{\partial(C_p v_p)}{\partial y}\right)+C_p\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)= \frac{3}{4}\frac{C_{Df}\rho_f C_p}{D_p\rho_p}\left|U-U_p\right|(u-u_p)+\frac{Q_p E_x C_p}{m_p} \tag{18}$$

$$v_p\left(\frac{\partial(C_p u_p)}{\partial x}+\frac{\partial(C_p v_p)}{\partial y}\right)+C_p\left(u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)= \frac{3}{4}\frac{C_{Df}\rho_f C_p}{D_p\rho_p}\left|U-U_p\right|(v-v_p)+\frac{Q_p E_y C_p}{m_p} \tag{19}$$

The first parenthesis in the two above equations are equal to zero according to the particle continuity equation. By eliminating the first terms of the above equations and omitting Cp from both sides of these two equations, they can be expressed in a general vector form as follows:

$$U_p.\nabla U_p=\frac{3}{4}\frac{C_{Df}\rho_f}{D_p\rho_p}\left|U-U_p\right|(U-U_p)+\frac{Q_p E}{m_p} \tag{20}$$

In the above equation Up and E are the particle velocity vector and electrical field strength vector respectively.
For complete simulation one needs electric field strength distribution, gas velocity distribution, particle charge and number concentration of particles. In the following section, the basic governing equations for computing the above parameters are introduced.
The particle continuity equation was modified by introducing the number frequency distribu-tion of particle size. As a result of this modification, the following equation for poly-disperse particles was obtained [9]:

$$\frac{\partial\left(u_P C_P(D_P)\right)}{\partial x}+\frac{\partial\left(v_P C_P(D_P)\right)}{\partial y}= \frac{\partial}{\partial x}\left(E_P\frac{\partial C_P(D_P)}{\partial x}\right)+\frac{\partial}{\partial x}\left(E_P\frac{\partial C_P(D_P)}{\partial x}\right)+S_G\,C_{P0}(D_P)-S_C C_{Pn}(D_P) \tag{21}$$

Where $C_P(D_P)$ is the number frequency distribution of the particle size ($C_P(D_P)dD_P$ is the number concentration of particles having diameters between D_P and (D_P+dD_P) and $C_{P0}(D_P)$ is the number frequency distribution of particle size at the entrance and may be considered as a log-normal distribution which can be obtained by the following relation:

$$C_{P0}(D_P) = \frac{C_{P0t}}{\sqrt{2\pi}D_P \ln(s)} \mathrm{EXP}\left[\frac{-(\ln\frac{D_P}{\overline{D}_P})^2}{2(\ln s)^2}\right] \tag{22}$$

It should be mentioned that other size distribution relations or even a numerical size distribution can be used for $C_{P0}(D_P)$ if necessary. More details about the parameters S_G and S_C can be found in reference [9].

The following section will explain methods used to evaluate various parameters used in Equations 16 and 17.

The following equation proposed by Tennekes and Lumly [10] was used to predict the gas diffusion coefficient, Ep:

$$E_P = C u' l \tag{23}$$

where C is a constant, u' is mean fluctuation velocity and l is characteristic length and usually defined in terms of ε, as follows [11]:

$$l = \frac{3u'^3}{2\varepsilon} \tag{24}$$

u' can be set equal to $\sqrt{k}$, where k is turbulence kinetic energy.

In order to predict fluid velocity distribution, the normal k-ε turbulent flow model was used. For a two-dimensional incompressible steady flow, considering electrical body force, the equations describing momentum and mass balance are as follows [12]:

$$\frac{\partial}{\partial x}\left[\rho_f u^2 - (\mu+\mu_t)\frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[\rho_f uv - (\mu+\mu_t)\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu_t \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu_t \frac{\partial v}{\partial x}\right) \tag{25}$$

$$\frac{\partial}{\partial x}\left[\rho_f uv - (\mu+\mu_t)\frac{\partial v}{\partial x}\right] + \frac{\partial}{\partial y}\left[\rho_f v^2 - (\mu+\mu_t)\frac{\partial v}{\partial y}\right] = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu_t \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial y}\left(\mu_t \frac{\partial v}{\partial y}\right) \tag{26}$$

$$\frac{\partial(\rho_f u)}{\partial x} + \frac{\partial(\rho_f v)}{\partial y} = 0 \tag{27}$$

$$\frac{\partial}{\partial x}(\rho_f uk)+\frac{\partial}{\partial y}(\rho_f uk)=\frac{\partial}{\partial x}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial y}\right)+\mu_t G-C_D\rho_f\varepsilon \quad (28)$$

$$\frac{\partial}{\partial x}(\rho_f u\varepsilon)+\frac{\partial}{\partial y}(\rho_f u\varepsilon)=\frac{\partial}{\partial x}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right)+\frac{\varepsilon}{k}(C_1\mu_t G-C_2\rho_f\varepsilon) \quad (29)$$

$$G=2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2 \quad (30)$$

$$\mu_t=C_\mu\frac{\rho_f k^2}{\varepsilon} \quad (31)$$

The parameters used in a normal k-ε model are as follows:

σ_k	σ_ε	C_μ	C_D	C_1	C_2
1.0	1.3	0.09	1.0	1.44	1.92

The governing equation for the electric field in ESP is the Maxwell equation, which is normally expressed in terms of voltage:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=-\frac{\int_{D_{P\,min}}^{D_{P\,max}} Q_P(D_P)C_P(D_P)\,dD_P}{\varepsilon_0} \quad (32)$$

where $Q_P(D_P)$ is the electric charge of particles having a diameter between D_P and D_P+dD_P, which is constant during the precipitation process for double-stage ESP. The electric field was found from the following equation:

$$\begin{cases} E_x=-\dfrac{\partial\phi}{\partial x} \\ E_y=-\dfrac{\partial\phi}{\partial y}\end{cases} \quad (33)$$

In single-stage ESPs, the charging and collecting of particles take place simultaneously, so in order to predict ESP's performance one needs the current-voltage characteristics. In double-stage ESPs polluted gas is passed through a high-voltage wire-plate channel for particles charging before entering the collection section. When the particle concentration of polluted gas is low, particles will be charged to saturation. In this study, in order to find $Q_P(D_P)$, it was assumed that the particles reached their saturation charge during the charging process. Thus the particle charge, $Q_P(D_P)$, was obtained by the following equation given by [13] for the saturation charge of a particle with the diameter D_P:

$$Q_P(D_P)=\frac{3\varepsilon_P}{1+\varepsilon_P}\pi\varepsilon_0 D_P^2 E_{chg} \quad (34)$$

Where E_{chg} is the mean electric field strength at the particle charging section.

Results

In order to examine the results, operation of the following ESP was considered as a case study:

The duct is 20 cm long and 3 cm wide, a block is set at 5 cm of entrance. The block is 1 cm high and 1 cm wide. For polluted air flowing through the duct, a temperature of 298 K, pressure of 1 atm, kinematic viscosity of 1.56×10^{-5} kg/m.s and air density of 1.18 kg/m^3 and for particles, relative permitivity of ε_P=3.8 and density of 2350 kg/m^3 were considered. The permitivity of free space is ε_0=8.854×10^{-12} F/m. The magnitude of the mean electric field strength at the charging cell is 2.5×10^5 V/m and inlet mean fluid velocity is 2 m/s. The grid mesh used for the numerical solution of fluid flow and the electrical field strength equation is shown in Figure 1. The control volumes form an 110×35 orthogonal grid. Figure 2 shows the contour plot of fluid stream lines for the specified used configuration. Figure 3 shows the contour plot of electric potential for the same configuration.

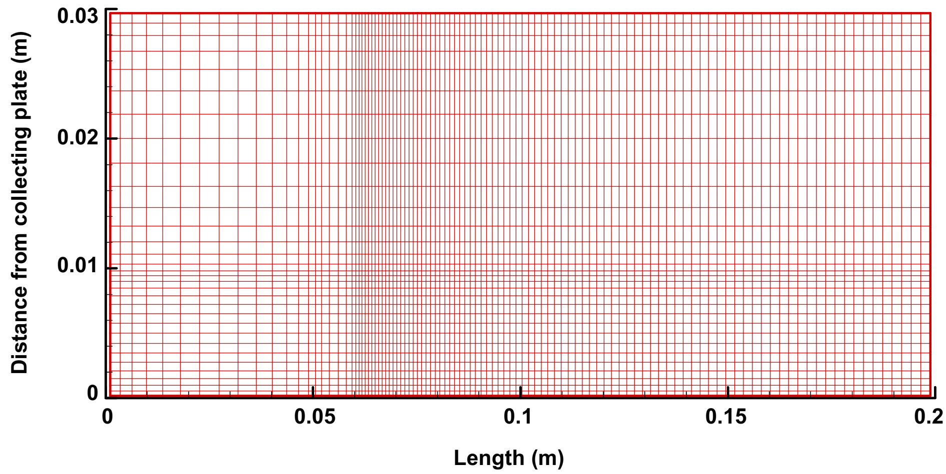

Figure 1. The grid mesh used

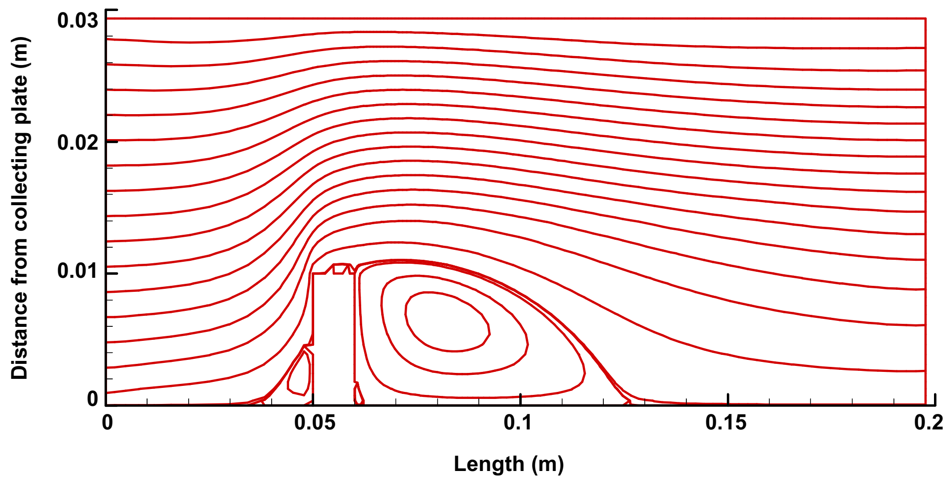

Figure 2. Contour plot of gas velocity stream lines, U_0=2m/s

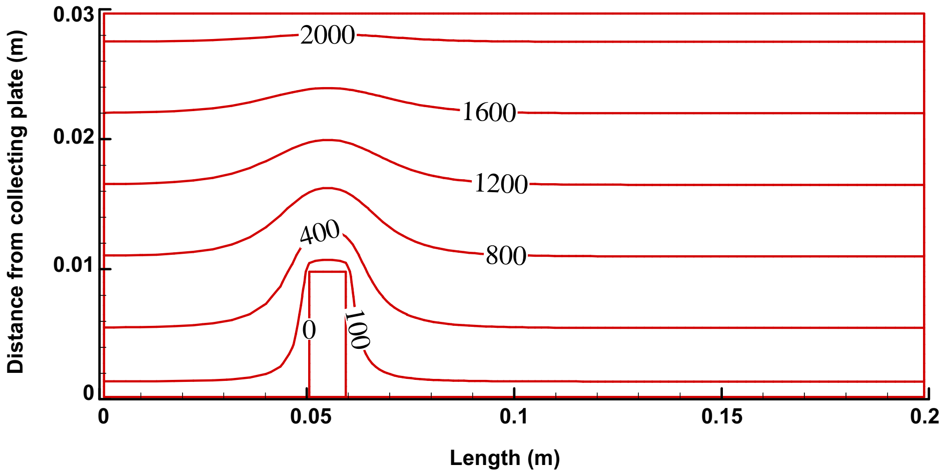

Figure 3. Contour plot of electric potential, applied voltage=2182 v

By now, the velocities calculated based on solving the particle momentum equation is called *the exact solution,* and *the error* of the velocity field means the mean difference of the velocities calculated by using any kind of explicit Eulerian method with those of a so-called exact solution. Figures 4 through 6 compare the particle velocity field calculated based on the equilibrium assumption (Equation 2) with the one obtained by the exact solution at different Stk and Es. These figures show that for higher Stk, the error will be more significant. By investigating the results shown in these figures, it was concluded that for Stk less than 0.001, the relative root mean square error is less than 1 percent. It can be interpreted so that the equailibrium assumption cannot be valid for Stk greater than 0.001. For high values of Stk the explicit Eulerian method based on the equilibrium assumption should not be used if one expects a reasonable calculation error. As mentioned above, the present explicit Eulerian method was optimized to minimize the calculation error of the particle velocity field. As expected, particle removal efficiencies predictted by using the present method are more accurate than those predicted by using the equilibrium assumption. Figures 7 through 10 show the capability of the present approach to calculate particle removal efficiencies accurately. As can be seen, the results have a much better agreement with the exact solution. Figures 11 through 13 show the variation of error with Stk for different Es. Apparently, using the present method produces less than the maximum 20% error compared to the maximum 120% error produced by using the equlibrium assumption.

Conclusion

The particle velocity field calculation for particle–laden flow through dust collector equipment was investigated to find the condition of the validity of the equlibrium assumption. The results are based on the simulation of particle removal in double-stage electrostatic precipitators. In exact solution, the particle mometum equation should be solved in order to find particle velocities. The dimensionless form of this equation is the same as that for any other dust collector. Thus the results can be valid for all particle collectors. By inspecting the results of simulation it was concluded that for Stk higher than 0.001, the relative root mean square error of particle velocity field is greater than 1 percent. Using the equilibrium assumption will be valid for cases having Stk up to 0.001. For Stk greater than 0.001, the time-consuming solution of the particle momentum equation should be made. In order to avoid solving this equation, a new explicit Eulerian method was developed. This method calcu-lates the particle velocity field explicitly for Stk greater than 0.001 with reasonable precision. Using this method reduces the fairly long computation time of the simulation of dust collector performance.

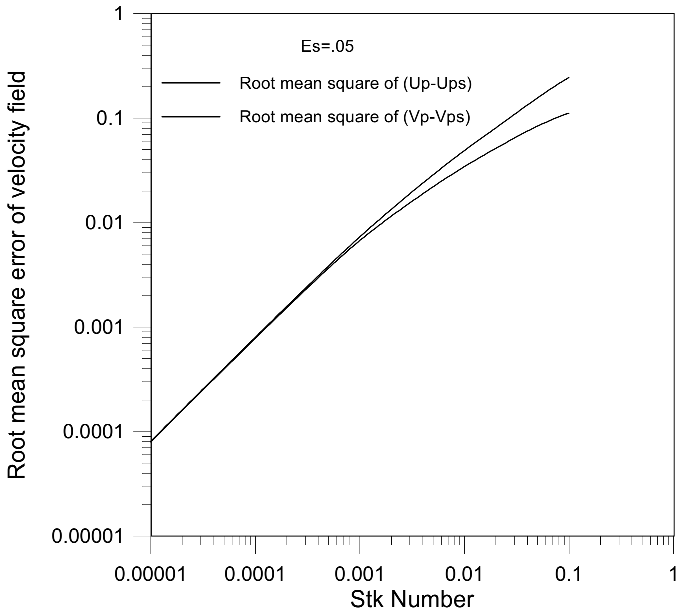

Figure 4. Comparison of particle velocity field error calculated at various Stokes numbers for Es=0.05

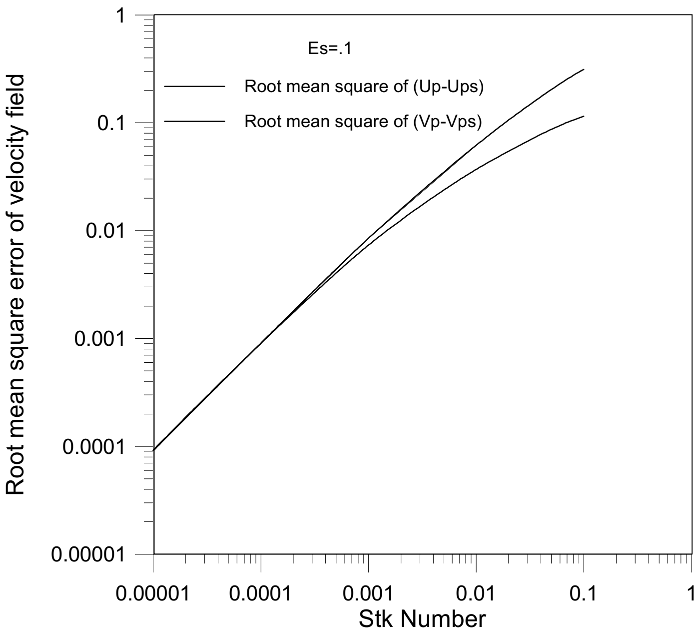

Figure 5. Comparison of particle velocity field error calculated at various Stokes numbers for Es=0.1

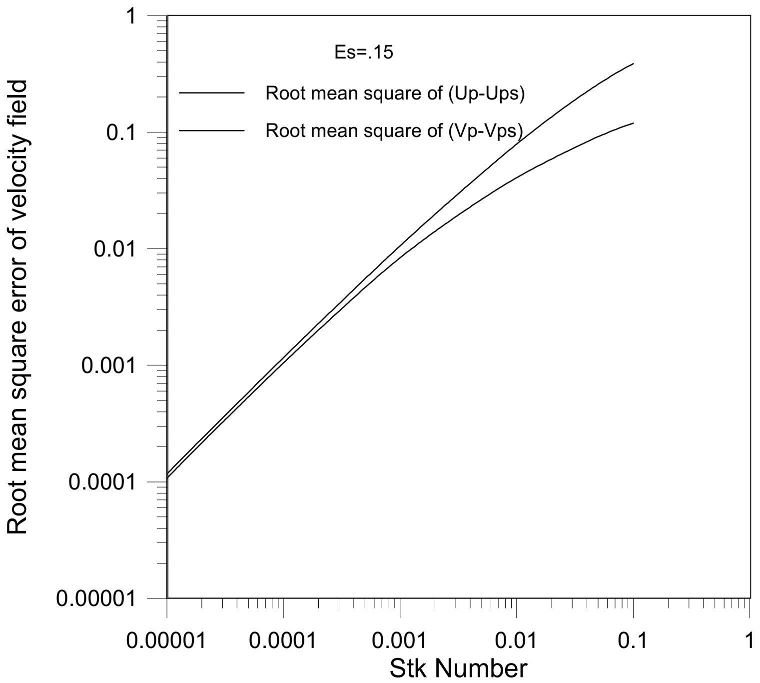

Figure 6. Comparison of particle velocity field error calculated at various Stokes numbers for Es=0.15

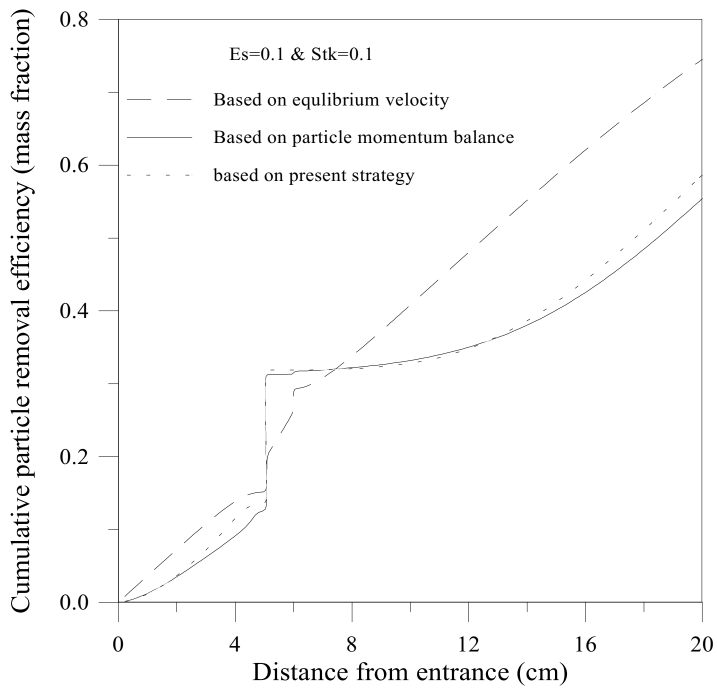

Figure 7. Variation of particle removal efficiency along ESP for Es=0.1 and Stk=0.1

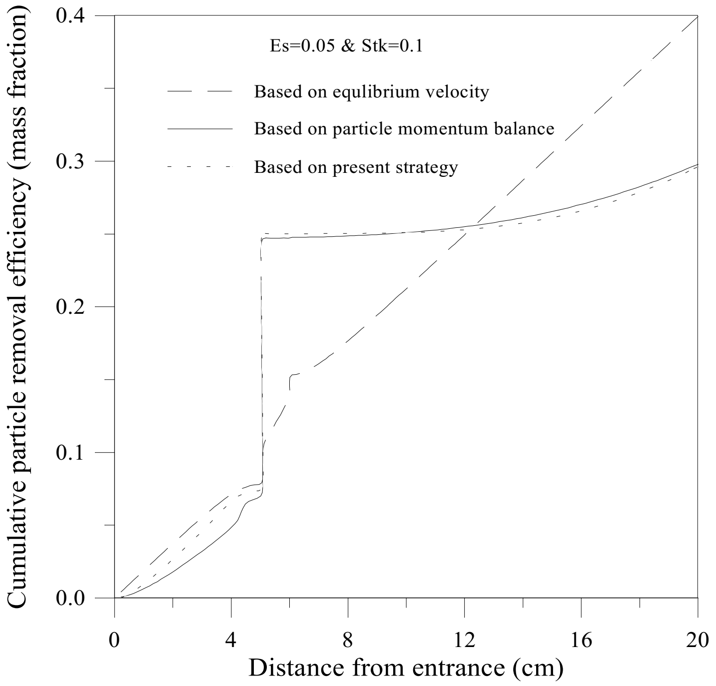

Figure 8. Variation of particle removal efficiency along ESP for Es=0.05 and Stk=0.1

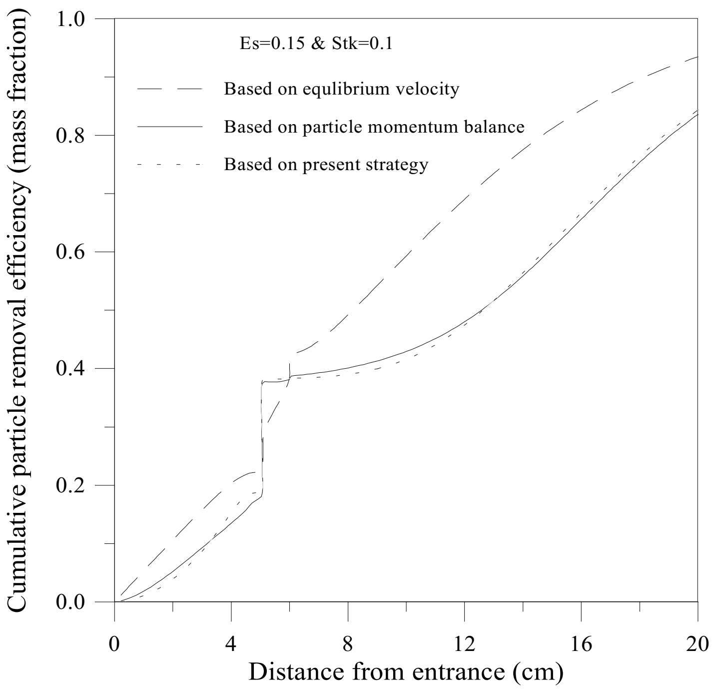

Figure 9. Variation of particle removal efficiency along ESP for Es=0.15 and Stk=0.1

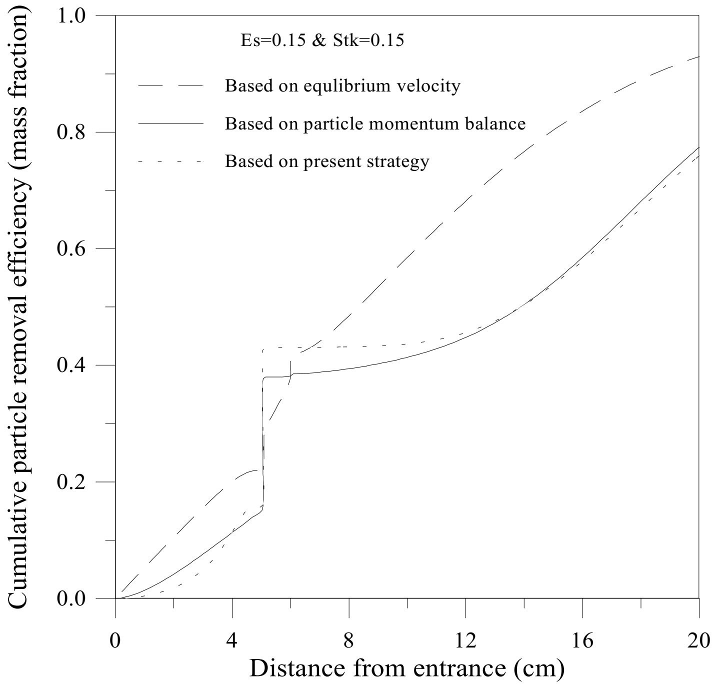

Figure 10. Variation of particle removal efficiency along ESP for Es=0.15 and Stk=0.15

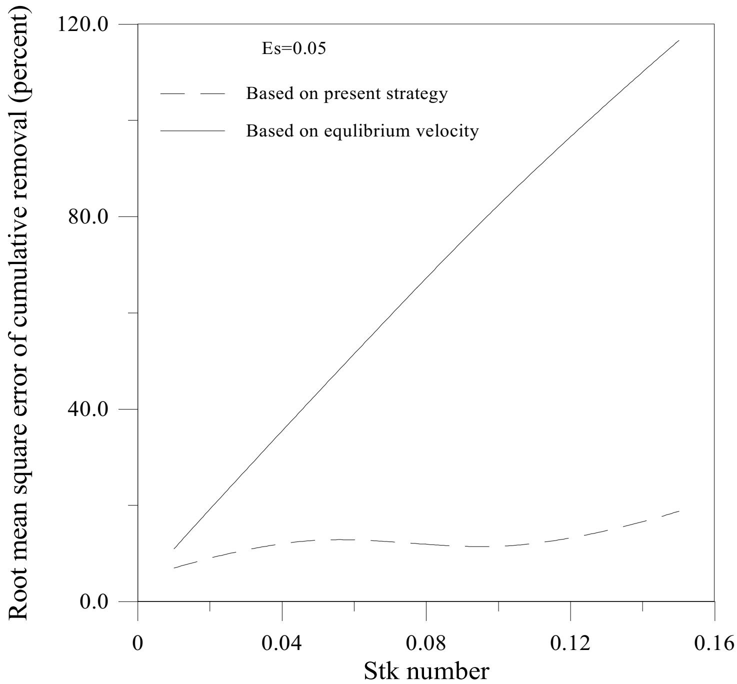

Figure 11. Variation of root mean squre error of cumulative removal efficiency with Stokes number for Es=0.05

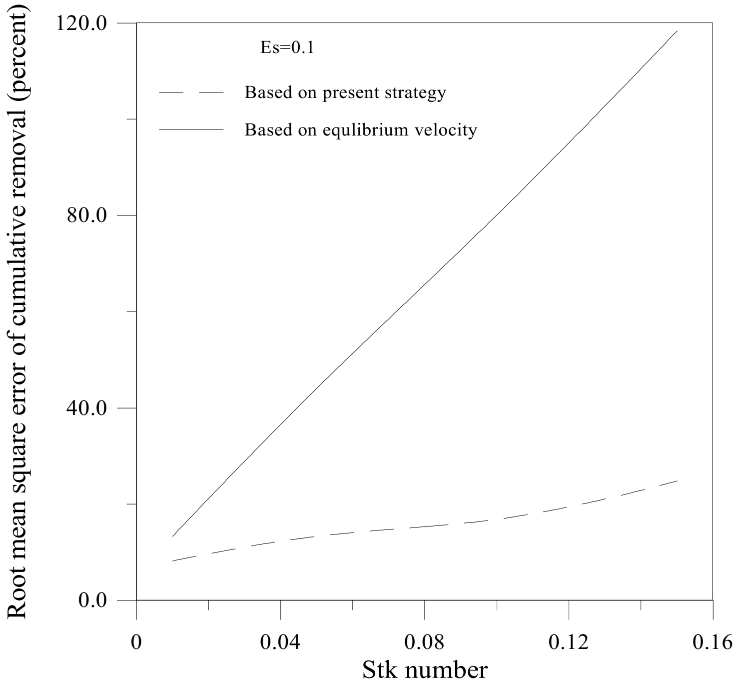

Figure 12. Variation of root mean squre error of cumulative removal efficiency with Stokes number for Es=0.1

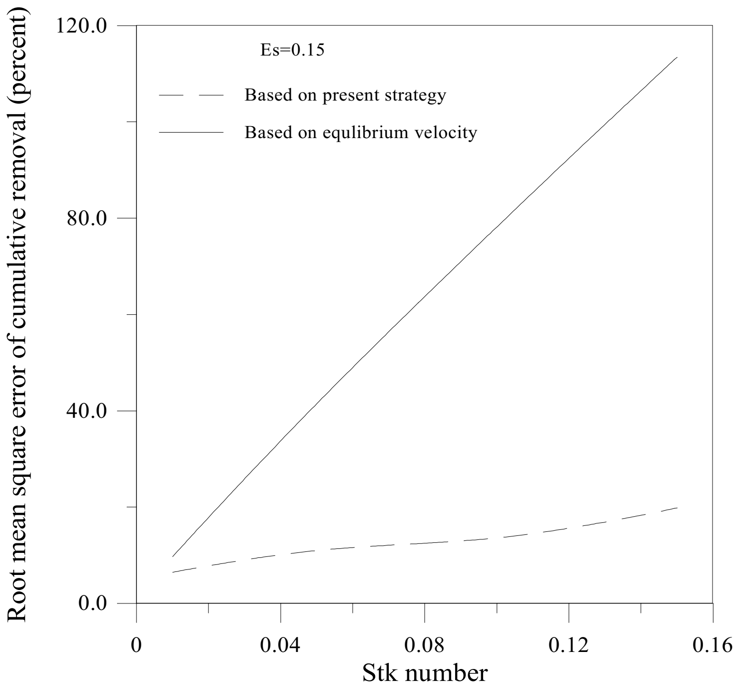

Figure 13. Variation of root mean squre error of cumulative removal efficiency with Stokes number for Es=0.15.

Nomenclature

C_C	Coninghum coefficient	(Dimensionless)
C_{Df}	Drag coefficient	(Dimensionless)
C_P	Number concentration of particles with diameter D_P	(No./m^3)
$C_P(D_P)$	Number frequency distribution of particle size	(No./m^4)
$C_{P0}(D_P)$	Number frequency distribution of particle size at entrance	(No./m^4)
D_P	Particle diameter	(m)
E	Electric field strength vector	(V/m)
E_0	(ϕ/L)	
E_x	x-component electric field strength	(V/m)
E_y	y-component electric field strength	(V/m)
E_{chg}	Electric field strength at particle charging section	(V/m)
E_p	Particle diffusivity coefficient	(m^2/s)
f	External force vector	
f^*	(f/E_0q_0)	
k	Turbulence kinetic energy	(J/s/kg)
L	characteristic length	(m)
m_p	mass of a particle	(kg)
P	Pressure	(kPa)
$Q_P(D_P)$	Electric charge of particles having diameter between D_P and D_P+dD_P	(C)
Q_P	Charge collected on a particle	(C)
Q_S	Saturation electric charge of particle	(C)
S_G	Source term in particle continuity	
S_C	Sink term in particle continuity	
U	Fluid velocity vector	
U^*	(U/u_0)	
Up	Particle velocity vector	
Up^*	(Up/u_0)	
U_{ps}	Equilibrium particle velocity vector	
U_{ps}^*	(U_{ps}/u_0)	
u_0	Maximum mean velocity of fluid	(m/s)
u	x-component fluid velocity	(m/s)
u_p	x-component particle velocity	(m/s)
u'	Mean fluctuation velocity	(m/s)
v	y-component fluid velocity	(m/s)
v_p	y-component particle velocity	(m/s)
x	x direction	(m)
x^*	(x/L)	(Dimensionless)
y	y direction	(m)
y^*	(y/L)	(Dimensionless)
w	Residual velocity vector	
w^*	(w/u_0)	

Greek letters

ρ	Fluid density	(kg/m^3)
ρ_p	Particle density	(kg/m^3)
ε	Turbulence dissipation rate	(J/s/kg)
ε_0	Permittivity of free air	(8.854×10^{-6} F/m)
ε_p	Relative permittivity	(Dimensionless)
ϕ	Electric potential	(V)
μ	Laminar viscosity	(kg/ms)
μ_t	Turbulent viscosity	(kg/ms)
ν	Kinematic viscosity	(kg/ms)

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