Iranian Journal of Chemical Engineering
Vol. 3, No. 2 (spring), 2006, IAChE

Robust Control of a High-Purity Distillation Column Using μ-Synthesis

K. Razzaghi and F. Shahraki[*]
Department of Chemical Engineering, University of Sistan and Baluchestan, Zahedan, Iran.

Abstract
Distillation control is a challenging undertaking given the inherent nonlinearity of the process, severe coupling present for dual-composition control and the sensitivity of disturbances. Among various distillation operations, management of the high-purity column poses a difficult control situation due to a number of characteristics of these systems, including strong directionality, ill-conditioning and strongly nonlinear behaviour. In this paper, a diagonal PID controller is designed and analysed for a high-purity distillation column by computing the structured singular value μ introduced by Doyle (1982). For this purpose, a structured uncertainty model has been developed which describes the dynamics of the column for the entire operating range. The achievable manageable performance is also defined in terms of the H_∞-norm of the weighted sensitivity function.

Keywords: *high-purity distillation, ill-conditioned plant, model uncertainty, μ-synthesis*

Introduction

Almost all chemical plants use distillation columns for the separation of substances according to their relative volatility. Distillation processes are among the largest energy consumers in chemical plants, therefore, tight control of the product compositions is fundamental for an economically optimal operation of the entire plant. Control systems for chemical processes are typically designed using an approximate, linear, time-invariant model of the plant. The actual plant may differ from the nominal model due to many sources of uncertainty, such as nonlinearity, the selection of low-order models to represent a plant with inherently high-order dynamics, inaccurate identification of model parameters due to poor measurements or incomplete knowledge, and uncertainty in the manipulative variables. Considering the differences between the actual plant and nominal model, it is necessary to insure that the control system will be stable and meet some predetermined performance criteria when applied to the actual plant. Many control design techniques have been applied to the high-purity distillation columns (e.g. Georgiou *et al*., 1988; Skogestad and Lundström, 1990; Christen *et al*., 1997). Some possible improvements for linear multivariable predictive control of high-purity distillation columns are proposed by

∗ - Corresponding author: E-mail: fshahraki@hamoon.usb.ac.ir

Trentacapilli *et al*. (1997) and a simple way of inserting a local model that contains part of the process nonlinearity into the controller is described also.

Another important aspect of distillation control design is the choice of a good configuration. In fact, poor control performance can result from the improper choice of manipulated/controlled variable pairing (Hurowitz *et al*., 2003). Some authors have even considered control configuration selection (Shinskey, 1984; Skogestad and Morari, 1987; Hurowitz *et al*., 2003), but there is no general agreement among these authors in choosing the best control configuration, however, a complete review in this field was undertaken by Skogestad *et al*. (1990).

The main works for selection of manipulated/controlled variable pairings have focused upon using controllability measures, such as the relative gain array (Bristol, 1966) and structured singular value μ (Doyle, 1982). The relative gain array (RGA) provides a steady-state measure of coupling in multivariable systems and can be used to evaluate the steady-state coupling of configurations. RGA is still the most commonly used tool for control structure selection for single-loop controllers. The structured singular value (SSV) approach provides necessary and sufficient conditions for robust stability and performance for the situation in which uncertainty occurs simultaneously and independently in various parts of the overall control system (e.g. input and output uncertainty) but the perturbation matrix is still norm-bounded. In formulating the SSV problem, use of the physically-based uncertainty description is important. Simplified models that predict gain and time constant changes as the process is perturbed over the expected operating regime can be used to characterise the uncertainty (McDonald *et al*., 1988).

The main objective of the robust multivariable control problem is to design a control law which maintains system response and error signals within prespecified tolerances despite the effects of uncertainty on the system. In this paper we will design a controller for a high-purity distillation column using μ framework for a plant model including input uncertainty. It should be noted that all of the results presented in this paper for μ-synthesis were computed using the MATLAB "μ-Analysis and Synthesis Toolbox" (Balas *et al*., 1993).

Theory

Process Description

Figure 1 shows a schematic of a binary distillation column that uses reflux and vapour boilup as manipulated inputs for the control of top and bottom compositions, respectively. This is referred as the *LV*-configuration (structure). Other possible dual-composition control configurations (such as *DB*-, *DV*- or double ratio (*L*/*D*) (*V*/*B*)-configuration) may be used, but the *LV*-configuration is chosen because this is the choice of manipulated inputs most commonly used in industrial practice (Skogestad and Morari, 1988a). Generally, for high reflux ratio columns, configurations other than the *LV*-configuration are preferred while for low to moderate reflux ratios, energy balance configurations were shown to perform better (Skogestad *et al*., 1990; Hurowitz *et al*., 2003). The distillation column model used in this paper is a high-purity column, referred to as the "column at operating point A" by Skogestad and Morari (1988b). Table 1 summarises the steady-state data of the model in detail. This is a good example for design of controllers for an ill-conditioned, high-purity distillation column, which is used by several researchers (e.g. Waller *et al*., 1994; Gjøsæter and Foss. 1997; Shin *et al*., 2000). The following simplifying assumptions are made: (1) binary separation, (2) constant relative volatility and (3) constant molar flows. We will add *uncertainty* to include the effect of any neglected flow dynamics when designing and analyzing the controller.

Table 1. Steady-state data for distillation column under study

Column data	
Relative volatility	$\alpha = 1.5$
No. of theoretical trays	$N_T = 40$
Feed tray (1 = reboiler)	$N_F = 21$
Feed composition	$z_F = 0.50$
Operating data	
Distillate composition	$y_D = 0.99$
Bottom composition	$x_B = 0.01$
Distillate to feed ratio	$D/F = 0.500$
Reflux to feed ratio	$L/F = 2.706$

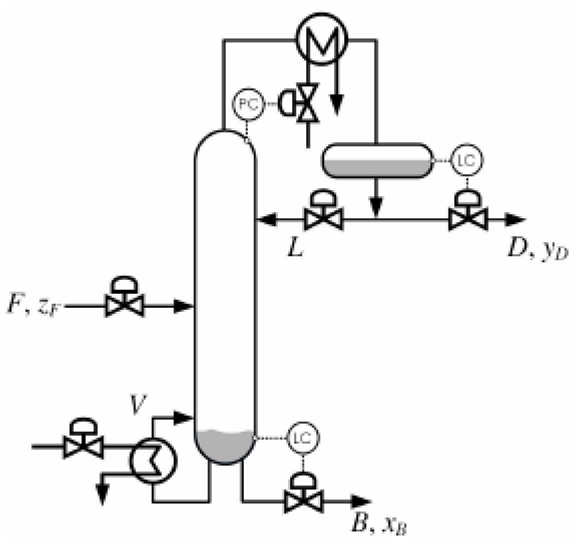


Figure 1. Schematic of a binary distillation column using the *LV*-configuration

A simple two time-constant dynamic model presented by Skogestad *et al*. (1990) is chosen as the basis for the controller design. The two time-constant model is derived assuming the flow and composition dynamics to be decoupled, and then the two separate models for the composition and flow dynamics are simply combined. The nominal model of the column is given by

$$d\,y_D = \frac{k_{11}}{1+\tau_1 s} dL + \left(\frac{k_{11}+k_{12}}{1+\tau_2 s} - \frac{k_{11}}{1+\tau_1 s} \right) dV,$$

$$d\,x_B = \frac{k_{21}}{1+\tau_1 s} g_L(s)\, dL + \left(\frac{k_{21}+k_{22}}{1+\tau_2 s} - \frac{k_{21}}{1+\tau_1 s} \right) dV. \quad (1)$$

This model includes the liquid flow dynamics and the differences between changes in external and internal flows. k_{ij} denotes the steady state gain around the normal operating point. $\tau_1 = 194\,\text{min}$ and $\tau_2 = 15\,\text{min}$ are the time constants associated with changes in external flows and internal flows, respectively. $g_L(s)$ expresses the liquid flow dynamics:

$$g_L(s) = \frac{1}{[1+(2.46/n)s]^n} \quad , \tag{2}$$

where n is the number of trays in the column (N_T −1). The model uses the *LV*-configuration where the manipulated inputs are the reflux flow rate, *L*, and the boilup rate, *V*. The process outputs are distillate composition, y_D, and the bottom product composition, x_B. However, this model is a high order model (41st order) and the resulting multivariable controller for the model will have 18 adjustable parameters which is difficult of implementation (Skogestad and Lundström, 1990). This nominal model of the column has a steady-state gain matrix as

$$G^{LV}(0) = \begin{pmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{pmatrix} \quad . \tag{3}$$

Since the structured singular value synthesis requires only steady-state data of the process, therefore we can neglect the effect of flow dynamics, but we will add uncertainty to compensate this neglected flow dynamics when we analyse the controller.

System Analysis

1. The Relative Gain Array (RGA)

The RGA of the matrix *G* (Bristol, 1966) is defined as

$$\Lambda(G) = G \times (G^{-1})^T \quad , \tag{4}$$

where × denotes the element-by-element multiplication. For 2×2 systems

$$RGA = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} = \begin{pmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{11} & \lambda_{11} \end{pmatrix}$$
$$and \quad \lambda_{11} = \frac{1}{1-\left(g_{12}g_{21}/g_{11}g_{22}\right)} \quad , \tag{5}$$

where g_{ij} s is an open-loop gain from the *j*th input to the *i*th output of the process. The RGA has been considered as an important MIMO system information for feedback control. Controllers with large RGA elements should generally be avoided, otherwise the closed-loop system is very sensitive to input uncertainty (Skogestad and Morari, 1987). In Figure 2, we plot the magnitude of the diagonal element of the RGA as a function of frequency for the plant. As it is evident, the column has large RGA values at low frequencies (steady-state), but at high frequencies λ_{11} is equal to one. This says that only based on the RGA plot, making a decision on the ill-conditionedness of the control problem may be misleading.

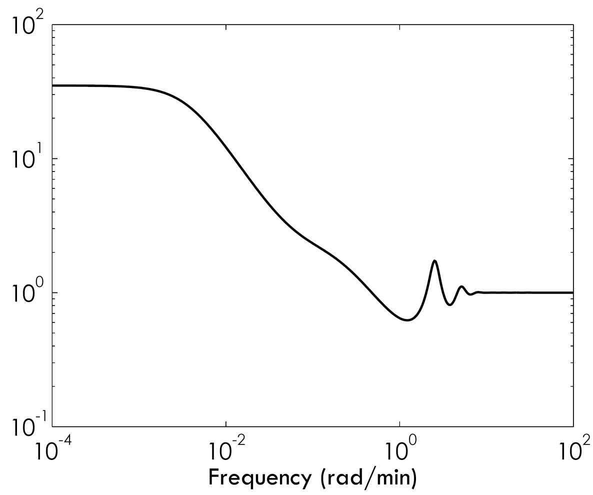


Figure 2. $|\lambda_{11}|$ as a function of frequency

2. Ill-conditionedness and process gain directionality

The common definition of an ill-conditioned plant is that it has a model with a large condition number (γ). The condition number is defined as the ratio between the largest and smallest singular values ($\overline{\sigma}/\underline{\sigma}$) of a process model. However, the condition number depends on the scaling of the process model. This problem arises from the scaling dependency of the Singular Value Decomposition (SVD). To eliminate the effect of scaling, the minimised condition number, $\gamma_{\min}$, is defined as the smallest possible condition number that can be achieved by varying the scaling (Grosdidier *et al*., 1985). In mathematical symbols

$$\gamma_{\min}(G) = \min_{S_1, S_2} \gamma(S_1 G S_2) \ , \tag{6}$$

where S_1 and S_2 are real, diagonal scaling matrices. The close relationship between $\gamma_{\min}$ and RGA is proposed by Grosdidier *et al.* (1985). For 2×2 systems

$$\gamma_{\min}(G) = \|\Lambda(G)\|_1 + \sqrt{\|\Lambda(G)\|_1^2 - 1} \ , \tag{7}$$

where the 1-norm of the RGA is defined as

$$\|\Lambda\|_1 = \max_j \sum_{i=1}^{m} |\lambda_{ij}| \ . \tag{8}$$

According to the above relationship, a 2×2 system with small RGA elements always has a small $\gamma_{\min}$. In particular, if $0 \le \lambda_{11} \le 1$ the minimised condition number is always equal to one. A process model with a large span in the possible gain of the model is said to show high directionality and a process model with the smallest singular value equal to the largest singular value is said to show no directionality. Waller *et al.* (1994) suggest redefined definition of process directionality. The definition divides the concept of process directionality into two parts. The minimised condition number is connected to stability aspects, whereas the condition number of a process model scaled according to the weight of the variables is connected to performance aspects. However, measuring process gain directionality may in some cases be problematic (Shahraki and Razzaghi, 2005). Figure 3 shows the largest and smallest singular values and condition number of the process model as a function of frequency.

The condition number of the process is about 10 times lower at high frequencies than at low frequencies (steady state). Figure 4(a) represents the values of γ and $\gamma_{\min}$ as a function of frequency. Values of γ and $\gamma_{\min}$ match each other from low to intermediate frequencies, but $\gamma_{\min}(G)$ approaches one at high frequencies. For 2×2 systems (Grosdidier *et al*., 1985)

$$\|\Lambda\|_1 - \frac{1}{\gamma_{\min}(G)} \le \gamma_{\min}(G) \le \|\Lambda\|_1 \ . \tag{9}$$

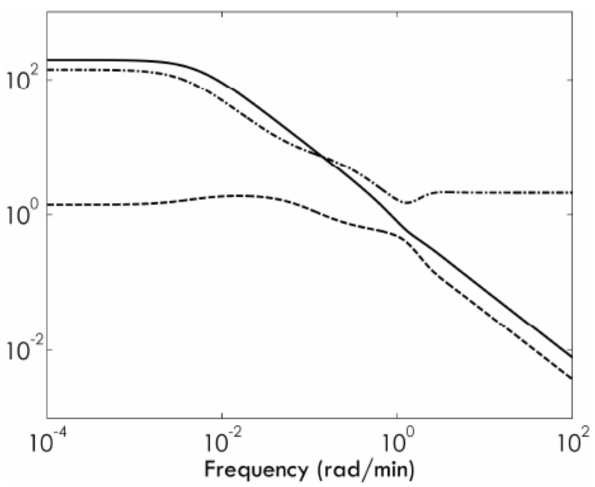


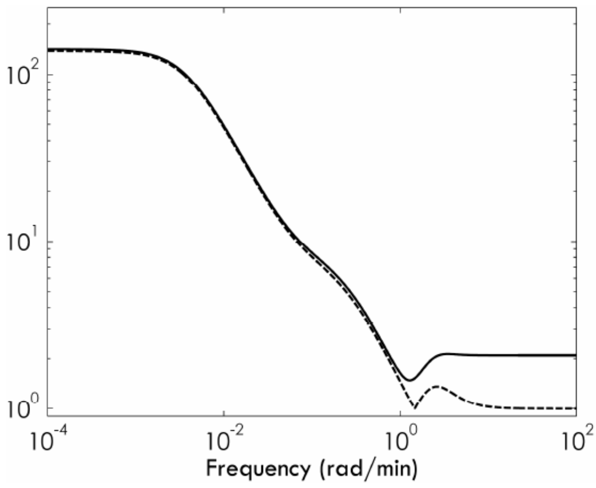
Figure 3. Singular values and condition number of the process. —— $\sigma_{\max}$, - - - $\sigma_{\min}$ and –·–· γ.

Consequently, for 2×2 systems the difference between these quantities is at most one and $\|\Lambda\|_1$ approaches $\gamma_{\min}$ as $\gamma_{\min} \to \infty$. Since $\|\Lambda\|_1$ is much easy to compute than $\gamma_{\min}$, it is the preferred quantity to use. In Figure 4(b), we plot $\gamma_{\min}$ and $\|\Lambda\|_1$ as a function of frequency. The value of $\gamma_{\min}$ at low frequencies is approximately twice $\|\Lambda\|_1$. At high frequencies, both $\gamma_{\min}$ and $\|\Lambda\|_1$ approach one (after $\omega = 20$ rad/min). This is in agreement with the result obtained from λ_{11}-vs.-frequency plot (Figure 2). Since $\gamma_{\min}$ is independent of scaling, therefore it is better to use $\gamma_{\min}$ instead of γ which is scale dependent.

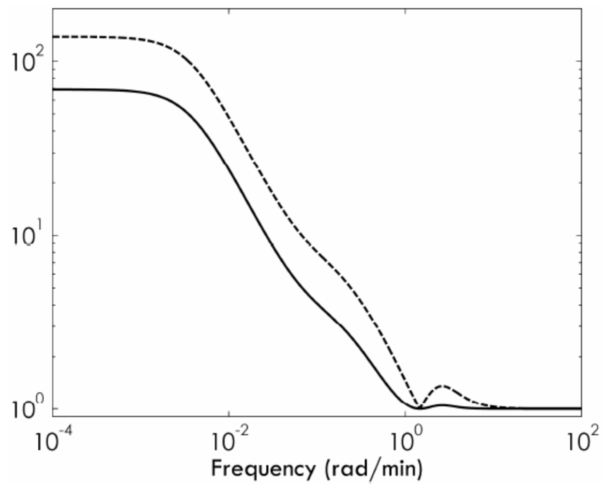
Uncertainty Models for Distillation Column

Figure 5(a) shows a block diagram of a distillation column with related inputs (u, d) and outputs (y, y_m). In Figure 5(b), we have added two additional blocks to Figure 5(a). One is the controller C, which computes the appropriate input u based on the information about the process, y_m, and the other block, Δ, represents the model uncertainty. The actual plant is different depending on Δ. Based on the measurements y_m, the objective of the controller C is to generate inputs u that keep the outputs y as close as possible to their set points in spite of disturbances d and model uncertainty Δ. The controller C is often nonsquare, as there are usually more measurements than manipulated variables. For the design of the controller C, information about the expected model uncertainty should be taken into account.

Usually, there are two main ways for adding uncertainty to a constructed model: additive and multiplicative uncertainty. Figure 5(c) represents additive uncertainty. In this case, the perturbed plant gain, G_p, will be $G+\Delta$. Figure 5(d) represents multiplicative uncertainty where the perturbed plant is equal to $G(\mathbf{I}+\Delta)$.



(a)



(b)

Figure 4. (a) Plots of γ and γmin as a function of frequency (—— γ and - - - $\gamma_{\min}$); (b) plots of $\|\Lambda\|_1$ and $\gamma_{\min}$ as a function of frequency (—— $\|\Lambda\|_1$ and - - - $\gamma_{\min}$)

Robust Performance and Robust Stability

The objective of using feedback control is to keep the controlled outputs (in our case y_D and x_B) close to their desired set points. What is meant by close is more precisely defined by the performance specifications. These performance requirements should be satisfied in spite of unmeasured disturbances and model-plant mismatch (uncertainty). Consequently, the ultimate goal of the controller design is to achieve robust performance (RP). The performance specification should be satisfied for the worst-case combination of disturbances and model-plant mismatch. To check for RP, we will use the structured singular value (μ). μ of a matrix N (denoted $\mu(N)$) is equal to $1/\bar{\sigma}(\Delta)$ where $\bar{\sigma}(\Delta)$ is the magnitude of the smallest perturbation needed to make the matrix $\mathbf{I}+\Delta N$ singular. $\mu(N)$ depends both on the matrix N and of the structure of the perturbation Δ (Skogestad and Morari, 1988a). Assume that G_p is the perturbed plant and is given by:

$$G_p = G(\mathbf{I}+\Delta_I) \quad \text{with} \quad \Delta_I = \text{diag}\{\Delta_i\}_{i=1,2} \ , \tag{10}$$

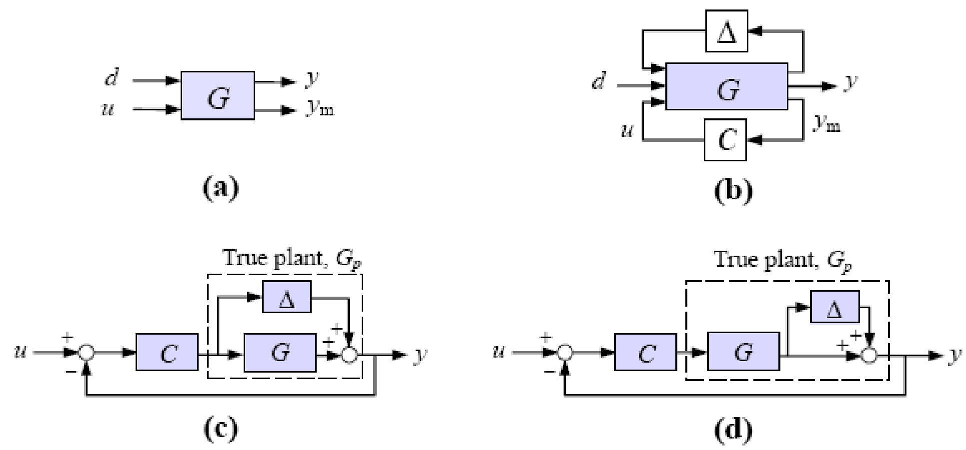


Figure 5. (a) Schematic representation of distillation column from system point of view (inputs and outputs); (b) general structure for studying any control problem; (c) additive uncertainty, $G_p = G + \Delta$; (d) multiplicative uncertainty, $G_p = G(\mathbf{I} + \Delta)$.

where Δ_I is input uncertainty weight and Δ_i is the relative uncertainty for input i ($i=1$ for reflux rate as manipulated input for control of distillate composition and $i=2$ for boilup as manipulated variable for control of bottom product composition). We will consider the case when the magnitude of this uncertainty is equal for both inputs. The robust stability can also be checked using μ. In this case

$$\text{RS} \Leftrightarrow \mu(N_{\text{RS}}) \le 1, \quad \forall \omega \in \Re \ , \tag{11}$$

where $N_{\text{RS}} = w_I CGS$. w_I is the input uncertainty weight and S is the sensitivity function of the plant and is defined as $S = (\mathbf{I}+GC)^{-1}$. To test for nominal performance (NP) we simply compute $\bar{\sigma}(w_P S)$ as a function of frequency. It should be less than one at all frequencies or equivalently

$$\text{NP} \Leftrightarrow \|w_P S\|_\infty < 1, \quad \forall \omega \in \Re \ . \tag{12}$$

To obtain robust performance (RP), the μ of the matrix N_{RP} should be less than one at all frequencies or

$$\text{RP} \Leftrightarrow \mu(N_{\text{RP}}) \leq 1, \quad \forall\omega \in \Re \ , \tag{13}$$

where

$$N_{\text{RP}} = \begin{pmatrix} w_I CSG & w_I CS \\ w_P SG & w_P S \end{pmatrix} . \tag{14}$$

and w_P is performance weight. μ is computed with respect to the structure $\text{diag}\{\Delta_I, \Delta_P\}$ where Δ_I is a 2×2 diagonal matrix and Δ_P is full 2×2 matrix.

Formulation of the Control Problem

Analysing the effect of uncertain models on achievable closed-loop performance and designing the controller to provide for optimal worst-case performance in the face of plant uncertainty are the main features that must be considered in robust control of an uncertain system. Skogestad *et al*. (1988) recommended a general guideline for modelling of uncertain systems. According to this, three types of uncertainty can be identified:

1. Uncertainty of the manipulated variables which is referred to as input uncertainty.
2. Uncertainty because of the process nonlinearity, and
3. Unmodelled high-frequency dynamics and uncertainty of the measured variables which is referred to as output uncertainty.

One source of uncertainty which always occurs in practice and generally limits the closed-loop's achievable performance of the process is input uncertainty which is the only source of uncertainty that is considered in this paper.

1. Uncertainty – The bounds for the relative errors of the column inputs *u* are modelled in the frequency domain by a multiplicative uncertainty with two frequency-dependent error bounds w_I. These two bounds are combined in the diagonal matrix $W_u = w_I \mathbf{I}$. In this case

$$\tilde{u}(j\omega) = \left[\text{I} + \Delta_I(j\omega) W_u(j\omega)\right] u(j\omega) \quad \text{with} \quad \left\| \Delta_I(j\omega) \right\|_\infty \leq 1 \ . \tag{15}$$

The value of the bound W_u is almost very small for low frequencies and increases substantially as we go to high frequencies. An error bound of 20% for the low frequency range is considered which is acceptable in a practical situation (Skogestad and Lundström, 1990). Higher errors must be assumed in the higher frequency range. Because of uncertain or neglected high-frequency dynamics or time delays, the input error exceeds 100%. The following weight is used as input uncertainty weight which is derived by use of the method presented by Skogestad and Lundström (1990):

$$w_I = 0.2 \frac{5s + 1}{0.5s + 1} \ . \tag{16}$$

The weight is shown graphically as a function of frequency in Figure 6.

2. Performance – For robust performance we must have

$$\overline{\sigma}(S_P) = \overline{\sigma}((I + G_p C)^{-1}) \leq \frac{1}{|w_P|} \tag{17}$$

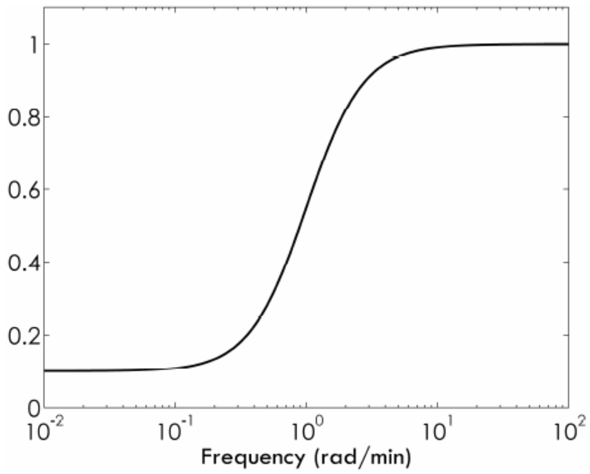


Figure 6. Input uncertainty weight $|w_u\ (\ j\omega)|$ as a function of frequency

for all possible plants, G_p. The performance weight is chosen as

$$w_P = 0.5\frac{10s+1}{10s}\ , \tag{18}$$

which is obtained by classical frequency domain specifications proposed by Skogestad and Lundström (1990).

Controller Design

Consider the feedback control diagram in Figure 5(b) where the effect of input uncertainty is taken into account by using a multiplicative uncertainty Δ. To analyse robust stability of the controller in the μ framework, one must recast the problem into the feedback loop diagram of Figure 7, where N is the linear system and Δ is a structured uncertainty. Suppose the peak value of $\mu(N)$ is β, then for all perturbation matrices Δ with the appropriate structure (i.e., any $\Delta \in \boldsymbol{\Delta}$), and satisfying $\max_\omega \overline{\sigma}(\Delta(j\omega)) < 1/\beta$, the perturbed system is stable (Balas *et al*., 1993). For the presented system, a diagonal PID controller based on internal model control (IMC) (Rivera *et al*., 1986) is designed. Optimal settings for single-loop PID controllers are found by minimising μ_{RP} (peak value of $\mu(N_{\mathrm{RP}})$). In addition, μ-optimal controller is designed, since it gives a good indication of the best possible performance of a controller.

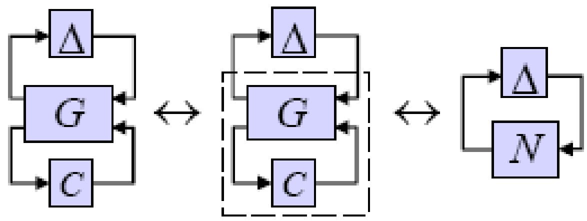


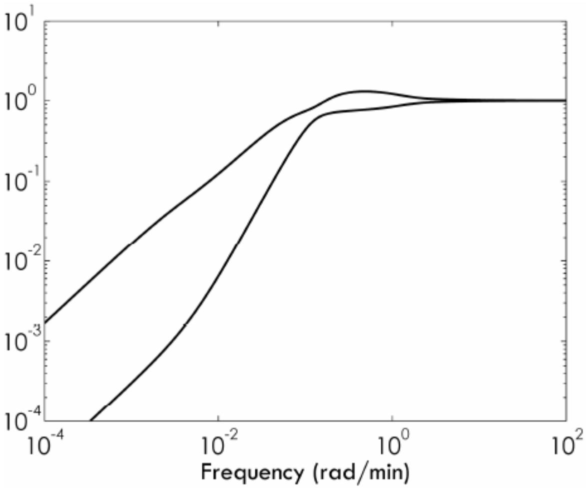
Figure 7. Transforming problem to general form for using in the μ framework

Simulation

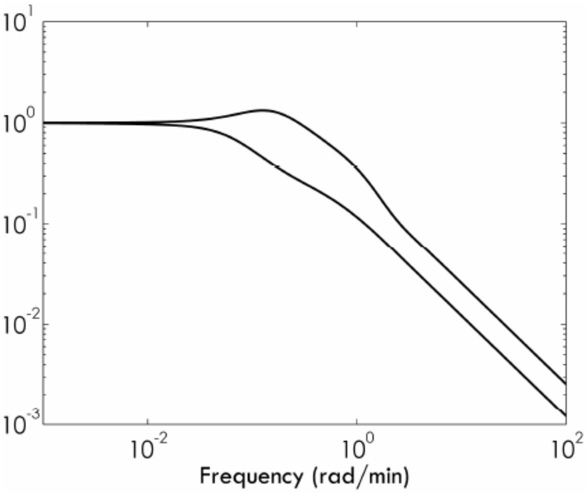
Simulations are carried out with the nonlinear model of the column and using single-loop controllers, which generally are insensitive to steady-state input errors (Skogestad and Morari, 1988a). In addition, input uncertainty is included to get a realistic evaluation of the controller.

Results

The plots of the singular values of the sensitivity functions $S = (\mathbf{I} + GC)^{-1}$ demonstrate good disturbance rejection properties, which indicate the closed-loop system is insensitive to uncertainties in inputs (Figure 8(a)). The tracking properties of this controller are also adequate, which is illustrated by plots of the complementary sensitivity function, $T = \mathbf{I} - S$ (Figure 8(b)). Up to the mid-frequency range, the singular values are close to one and the maximum of the upper singular values is slightly greater than one.



(a)



(b)

Figure 8. Singular values of the closed-loop system. (*a*) Sensitivity function; (*b*) complementary sensitivity function

PID-Controller Tuning

Table 2 summarizes the PID controller setting that is used for the column. Figure 9 shows μ-plots of the controller. From a maximum peak-value point of view, it is seen that both robust and nominal performance plots are less than one which satisfy the specified criteria.

Simulations

Simulations of a set-point change in y_D using the PID- and μ-optimal controllers are shown in Figures 10 and 11, respectively. As it is seen, the introduced uncertainty do not seriously affect the performance of the μ-optimal controller, while for the PID-controller, the effect of uncertainties is more rather the μ-optimal controller. It should be noted that the reference signal is filtered by a prefilter with a time constant of 5 min. In Figure 12, the closed-loop response for both controllers is shown simultaneously. As the figure shows, the PID controller needs considerably more times to reach steady-state than the μ-optimal controller. This is due to the high μ-value of NP at lower frequencies for the PID controller compared with the μ-optimal controller (0.661 for PID controller and 0.530 for μ-optimal controller).

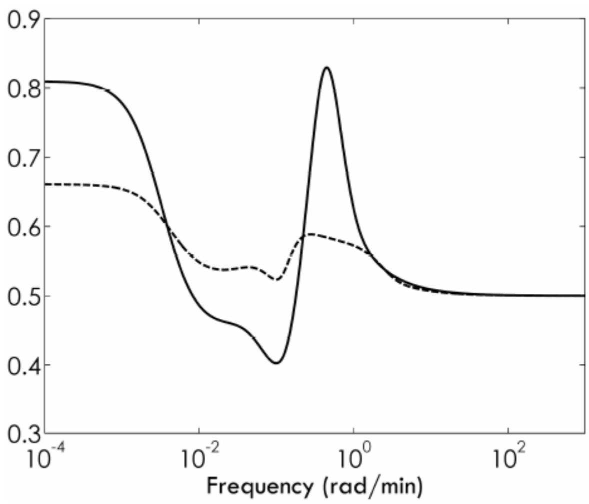


Figure 9. μ plots for PID controller. ▬▬ Robust Performance; - - - nominal Performance

Discussion

The structured singular value is used to investigate the robust performance of the PID controller. The control problem formulation used in this paper utilizes weighted input uncertainty. The inclusion of input uncertainty prevents the control system from becoming sensitive to disturbances, as may happen with inverting controllers.

Table 2. μ-optimal PID-controller tunings

	k	τ_I (min)	τ_D (min)
Top composition control loop	0.63	6.30	1.07
Bottom composition control loop	0.48	5.12	0.87

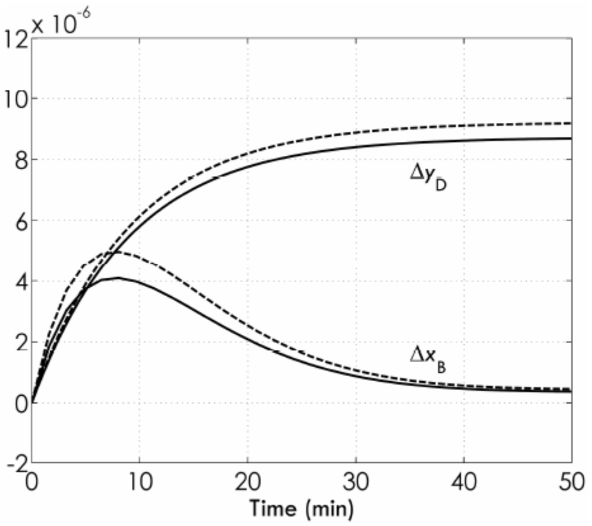


Figure 10. Closed-loop response to small set-point change in y_D (PID controller). ▬▬ No uncertainty; ‒ ‒ ‒ 10% uncertainty on input

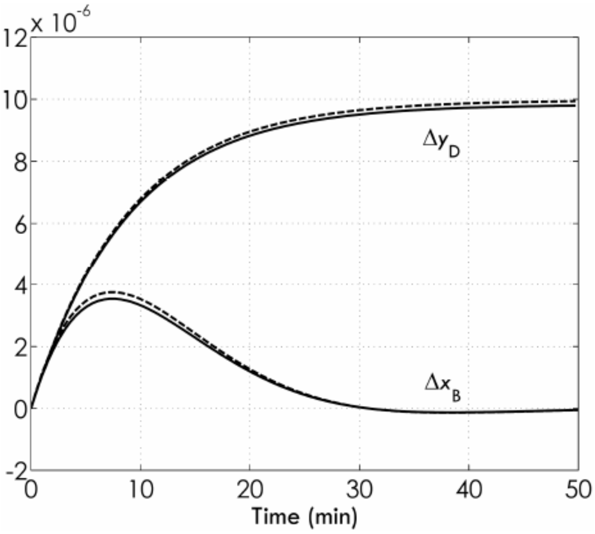


Figure 11. Closed-loop response to small set-point change in y_D (μ-optimal controller). ▬▬ No uncertainty; ‒ ‒ ‒ 10% uncertainty on input

The solution of the problem leads to the inequality of Eq. (13). The numerical solution of this design task is difficult. At present, there is no direct method to synthesise a μ-optimal controller, however, combination of μ-analysis and H_∞-synthesis which called μ-synthesis or DK-iteration, often yields good results. This algorithm has two drawbacks. Firstly, the algorithm cannot guarantee convergence, and secondly, the algorithm requires a scaling of the plant in each iteration step, which increases the order of the plant.

The μ analysis advantageously avoids dealing explicitly with the bad condition of the plant. With the μ-approach, the upper limit for the bandwidth of the control system is provided by the uncertainty model, whereas the lower limit is a matter of optimization. μ-synthesis is ideally suited to deal with complex uncertainty models which take into account such aspects as various operating points. A difficulty that one may encounter in the synthesis of the controller is high computation time, because the μ approach requires scaling in each iteration.

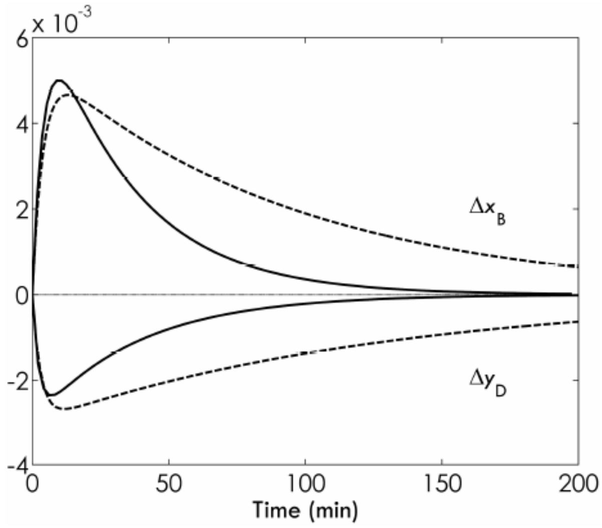


Figure 12. Closed-loop response to a 20% increase in feed flow rate (including input uncertainty). ▬▬ μ-optimal controller; ‒ ‒ ‒ PID controller

In this paper, the *LV*-configuration is used. The use of this configuration for columns with a high condition number may be doubtful, but under special considerations, this configuration may yield acceptable performance. In addition, severe interactions and poor control often reported with the *LV*-con-figuration may be almost eliminated if the loops are tuned sufficiently tight. However, this does not imply that the *LV*-configuration is the best structure to use. Shinskey (1984) showed that the use of the (*L*/*D*)(*V*/*B*)-configuration is probably better in most cases, and in particular for columns with large reflux.

Conclusion

For the design of the μ-optimal controller, a structured uncertainty model has been developed which describes the dynamics of a high-purity distillation column for the entire operating range. The structured singular value (μ) was used as a tool for evaluating the achievable control performance and performance was defined in terms of the H_∞-norm of the weighted sensitivity function, $w_P S$. A diagonal PID controller was found to be robust with respect to model-plant mismatch; also simulation results demonstrated good disturbance rejection capability of the designed controller.

Nomenclature

B	Bottom product rate
C	Controller
d	Disturbance
D	Distillate rate
F	Feed rate
G	Process gain matrix
$\mathbf{I}$	Identity matrix
k	Controller gain
L	Reflux rate
n	Number of trays in the column ($N_T - 1$)
N	Linear system matrix
N_T	Number of theoretical trays
N_F	Feed tray
S	Sensitivity function
u	Process input (manipulated variable)
V	Vapour boilup rate
w	Weighting function
x_B	Bottom composition
y	Process output (controlled variable)
y_D	Distillate composition
y_m	Measured variable
z_F	Feed composition
$\|\cdot\|_1$	1-norm of (·)
$\|\cdot\|_\infty$	∞-norm of (·)

Greek letters

α	Relative volatility
β	Peak value of μ
γ	Condition number
Δ	Uncertainty matrix
Δ	Set of all possible uncertainties
λ	Element of the RGA
Λ	Relative gain array (RGA)
μ	Structured singular value
σ	Singular value
τ	Time constant
ω	Frequency

Subscripts

D	Derivative
I	Input, Integral
min	Minimised
p	Perturbed
P	Performance

Superscript

T	Transpose

References

1. Balas, G. J., Doyle, J. C., Glover, K. A., Packard, K., and Smith, R., "The μ-Analysis and Synthesis Toolbox," The MathWorks Inc., Natick, MA., (1993).
2. Bristol, E. H., "On a new measure of interactions for multivariable process control," *IEEE Transactions on Automatic Control*, **AC-11**, 133 (1966).
3. Christen, U., Musch, H. E., and Steiner, M., "Robust control of distillation columns: μ-vs. H_∞-synthesis," *Journal of Process Control*, **7**(1), 19 (1997).
4. Doyle, J. C., "Analysis of feedback systems with structured uncertainties," *IEE Proceedings*, **129**(6), 242 (1982).
5. Georgiou, A., Georgakis, C., and Luyben, W. L., "Nonlinear dynamic matrix control for high-purity distillation columns," *A.I.Ch.E. Journal*, **44**(1), 106 (1988).
6. Gjøsæter, O. B., and Foss, B. A., "On the use of diagonal control versus decoupling for ill-conditioned processes," *Automatica*, **33**(3), 427 (1997).
7. Grosdidier, P., Morari, M., and Holt, B. R., "Closed-loop properties from steady-state gain information," *Industrial and Engineering Chemistry Research*, **24**, 221 (1985).
8. Hurowitz, S., Anderson, J., Duvall, M., and Riggs, J. B., "Distillation control configuration selection," *Journal of Process Control*, **13**, 357 (2003).
9. McDonald, K. A., Palazoglu, A., and Bequette, B. W., "Impact of model uncertainty descriptions for high-purity distillation control," *A.I.Ch.E Journal*, **34**(12), 1996 (1988).
10. Rivera, D. E., Morari, M., and Skogestad, S., "Internal model control: 4. PID cont-roller design," *Industrial and Engineering Process Design and Development*, **25**, 252 (1986).
11. Shahraki, F., and Razzaghi, K., "On a problem regarding process gain directionality error measures," *The 55th Canadian Chemical Engineering Conference*, Toronto, Ontario, December 16-19, (2005).
12. Shin, J., Seo, H., Han, M., and Park, S., "A nonlinear profile observer using tray temperatures for high-purity binary distillation column control," *Chemical Engineering Science*, **55**, 807 (2000).
13. Shinskey, F. G., "Distillation Control," 2nd ed., McGraw-Hill, New York, (1984).
14. Skogestad, S., and Lundström, P., "Mu-optimal *LV*-control of distillation columns," *Computers and Chemical En-gineering*, **14**(4/5), 401 (1990).
15. Skogestad, S., and Morari, M., "Control configuration selection for distillation columns," *A.I.Ch.E. Journal*, **33**(10), 1620 (1987).
16. Skogestad, S., and Morari, M., "*LV*-control of a high-purity distillation column," *Chemical Engineering Science*, **43**(1), 33 (1988a).
17. Skogestad, S., and Morari, M., "Understanding the dynamic behavior of distillation columns," *Industrial and Engineering Chemistry Research*, **27**, 1848 (1988b).
18. Skogestad, S., Morari, M., and Doyle, J. C., "Robust control of ill-conditioned plants: high-purity distillation," *IEEE Trans-actions on Automatic Control*, **12**, 1092 (1988).
19. Skogestad, S., Lundström, P., and Jacobsen, E. W., "Selecting the best distillation control configuration," *A.I.Ch.E. Journal*, **36**(5), 753 (1990).
20. Waller, J. B., Sågfors, M. F., and Waller, K. E., "Ill-conditionedness and process directionality–The use of condition numbers in process control," *Proceedings of IFAC Symposium*, Kyoto, Japan, 465 (1994).