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**Research note**

# Magnetohydrodynamic (MHD) Plane Poiseuille Flow With Variable Viscosity and Unequal Wall Temperatures

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## Abstract

*The plane Poiseuille flow with unequal wall temperatures of an incompressible fluid having temperature dependent viscosity in the presence of transverse magnetic field is studied. The coupled differential equations of momentum and energy are solved numerically by RKF45 (Runge-Kutta-Fehlberg fourth-fifth) method. Numerical results for the dimensionless velocity profiles, the temperature profiles and the heat transfer coefficient are presented and discussed graphically for various parameters. The study provides quantitative information of interest; in general, it is observed that the maximum velocity does not occur in the middle of the channel but moves towards the upper wall as the magnetic field increases. The temperature and heat transfer coefficient increases with the increasing value of magnetic field and EPr.*

**Keywords:** *Poiseuiller Flow, Variable Viscosity, Unequal Wall Temperature, Numerical Study*

## 1. Introduction

The magnetohydrodynamic (MHD) flow between two parallel walls is a classical problem whose solution has many applications in magnetohydrodynamic power generators, magnetohydrodynamic pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of oil and fluid droplets and sprays, etc. The subject of magnetohydrodynamic is largely perceived to have been initiated by Swedish electrical engineer Hannes Alfvén [1] in 1942. If an electrically conducting fluid is placed in a constant magnetic field, the motion of the fluid induces currents which create forces on the fluid. The production of these currents has led to the design of, among other devices, the magnetohydrodynamic generators for electricity production. The equations which describe magnetohydrodynamic flow are a combination of continuity equation and Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. The governing equations are differential equations that have to be solved either analytically or numerically.

Nahme [2] considered the plane Couette flow for fluid having temperature dependent

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viscosity, Hausenblas [3] considered the plane Poiseuille flow taking a simplified form of the viscosity and temperature relation keeping both walls at the same temperature, the same problem was considered by Bansal and Jain [4] taking the walls at unequal temperature. Shercliff [5] studied the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Drake [6] considered flow in a channel due to periodic pressure gradient and solved the resulting equation by separation of variables method. Singh & Ram [7] considered laminar flow of an electrically conducting fluid through a channel in the presence of a transverse magnetic field under the influence of a periodic pressure gradient and solved the resulting differential equation by the method of Laplace transform. Ram *et al*. [8] have analyzed hall effects on heat and mass transfer flow through porous media. Shimomura [9] discussed magnetohydrodynamics turbulent channel flow under a uniform transverse magnetic field. Singh [10] considered steady magnetohydrodynamic fluid flow between two parallel plates. Kazuyuki [11] discussed inertia effects in two dimensional magnetohydrodynamic channel flow and Al-Hadhrami [12] considered flow of fluids through horizontal channels of porous materials and obtained velocity expressions in terms of the Reynolds number. Ganesh [13] studied unsteady magnetohydrodynamic Stokes flow of a viscous fluid between two parallel porous plates. They considered fluid being withdrawn through both walls of the channel at the same rate.

In this paper we consider the problem of plane Poiseuille flow with unequal wall temperatures of an incompressible electrically conducting fluid under the influence of transverse magnetic field. The boundary layer equations governing the motion are studied by Bansal and Jain [4], in the absence of magnetic field. In the present study, the influence of transverse magnetic field on plane Poisenille flow with unequal wall temperature of an incompressible electrically conducting fluid is investigated. The governing boundary layer equation of momentum and energy are solved numerically using Runge-Kutta-Fehlberg Fourth-Fifth order method and the effect of various parameters is presented and discussed.

## 2. Formulation of the problem

Consider a steady two-dimensional laminar electrically conducting flow with variable viscosity between two parallel walls, taking x-axis along the central line of the channel and y-axis is perpendicular to it. A magnetic field of uniform strength $B_o$ is applied in the y-direction, which produces the magnetic field in the x-direction. We consider the case of a short circuit problem in which the applied electric field is $E = 0$, and also assume that the induced magnetic field is small compared to the external applied magnetic field $B_o$. This implies a small magnetic Reynolds number. The boundary layer equations governing the motion are (Bansal and Jain [4]):

$$\frac{d}{dy}\left(\bar{\mu}\frac{d\bar{u}}{dy}\right)-\sigma_e B_0^2\bar{u}=\frac{dp}{dx}. \qquad (1)$$

$$k\frac{d^2\bar{T}}{dy^2}+\bar{\mu}\left(\frac{d\bar{u}}{dy}\right)^2=0 \qquad (2)$$

Along with the boundary conditions are:

$$y = h : \bar{u} = 0, \bar{T} = \bar{T}_1$$
$$y = -h : \bar{u} = 0, \bar{T} = \bar{T}_0 \; (Where\ \bar{T}_1 > \bar{T}_0) \qquad (3)$$

Where $2h$ is the distance between the walls, and the motion is due to the constant pressure gradient $dp/dx$ along the axis of the channel. Let us introduce the following non-dimensional quantities:

$$u = \frac{\bar{u}}{u_m}, \eta = \frac{y}{h}, \mu = \frac{\bar{\mu}}{\mu_0}, T = \frac{\bar{T} - \bar{T}_0}{\bar{T}_1 - \bar{T}_0} \qquad (4)$$

Then equations (1) and (2) take the forms:

$$\frac{d}{d\eta}\left(\mu \frac{du}{d\eta}\right) - Mu = -2 \qquad (5)$$

$$\frac{d^2T}{d\eta^2} + \mu E\ Pr\left(\frac{du}{d\eta}\right) = 0 \qquad (6)$$

And the corresponding boundary conditions reduce to:

$$\eta = 1 : u = 0, T = 1$$
$$\eta = -1 : u = 0, T = 0 \qquad (7)$$

Where $Pr = \mu_0 C_p / k$ (Prandtl number), and $E = u_m^2 / C_p(\bar{T}_1 - \bar{T}_0)$ (Eckert number) where $u_m (= -\frac{h^2}{2\mu_0}\frac{dp}{dx})$ is the maximum velocity in the middle of the channel in the plane Poiseuille flow with constant fluid properties (Schilichting [14]), $\mu_0$ is the viscosity of the fluid at temperature $\bar{T}_0$ and $M = \sigma_e B_0^2 h^2 / \mu_0$ is the non-dimensional magnetic parameter.

## 3. Numerical solution

The differential equations (5) and (6) subject to boundary condition (7) are solved taking an empirical relation between viscosity and temperature $1/\mu = 1 + \alpha T$ (Hausenblas [3]) numerically using Runge-Kutta-Fehlberg fourth-fifth order method. To solve these equations we adopted symbolic algebra software Maple. Maple uses the well known Runge-Kutta-Fehlberg Fourth-fifth order (RFK45) method to generate the numerical solution of a boundary value problem. The effect of various parameters on velocity profile, temperature profile and the coefficient of heat transfer in terms are shown in Figs. 1 to 5.

## 4. Results and discussion

When the walls of the channel are at different temperatures, numerical computations are performed for various values of the physical parameters involved in the equations viz. the magnetic parameter M, $\alpha$ (where $\alpha = \frac{1}{2}(\frac{1}{\mu} - 1))$ and *EPr* (where *E* is the Eckert number and *Pr* is the Prandtl number). The calculated results are presented in Figs 1-5 to understand the effects of parameters on the flow and temperature field.

The impacts of magnetic parameter M on the velocity and temperature profiles are very significant from a practical point of view. Figs. 1 and 2 show the effects of magnetic parameter (M) on the velocity profile for the different values of α respectively. It is observed that in the present case maximum velocity does not occur in the middle of the channel but moves towards the upper wall as the value of M and $\alpha$ increases. Moreover, the rise in magnitude of the velocity is quite significant in the present case showing that the volume rate of flow at a section increases with increase in M and $\alpha$. This happens due to the Lorentz force arising from the interaction of magnetic and electric field during the motion of the electrically conducting fluid.

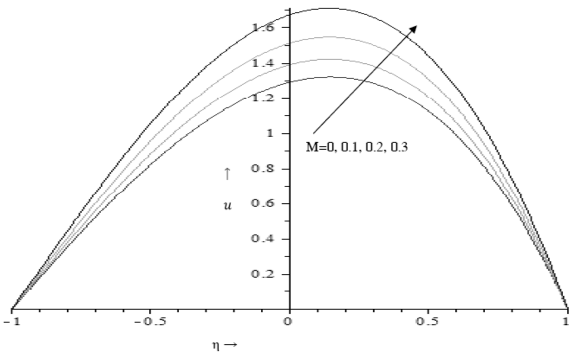


**Figure 1.** Velocity profile, against the perpendicular distance from central line, for different values of magnetic parameter M (when α=0.5).

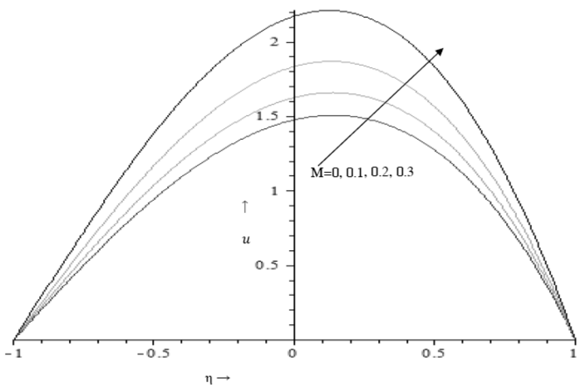


**Figure 2.** Velocity profile, against the perpendicular distance from central line, for different values of magnetic parameter M (when α=0.7).

On the other hand, Figs. 3 and 4 show the effects of magnetic parameter (M) on the temperature profile for the different values of *EPr* respectively. From the plot it is observed that temperature increases with the increasing value of M and *EPr*. This change in the temperature distribution leads to an important conclusion, that in the present case the transfer of heat at the lower wall will be more when compared with the constant viscosity case.

The variation of the temperature gradient $-T'(0)$ which is significant in evaluating the rate of heat transfer is presented in Fig. 5, against magnetic parameter M for the different values. It is observed that $-T'(0)$ increases with the increasing value of M and *EPr*. This heat transfer is very important in production engineering to improve the quality of the final product.

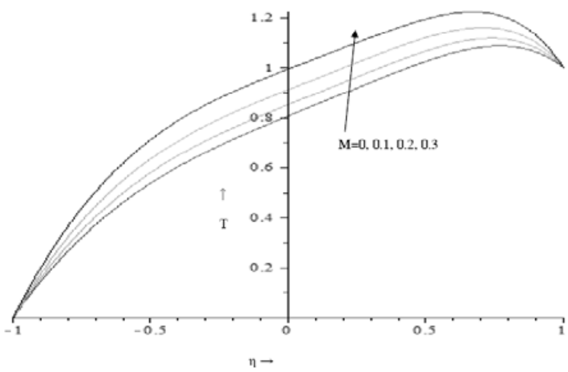


**Figure 3.** Temperature profile against the perpendicular distance from central line for different values of magnetic parameter M (when *EPr* =0.7).

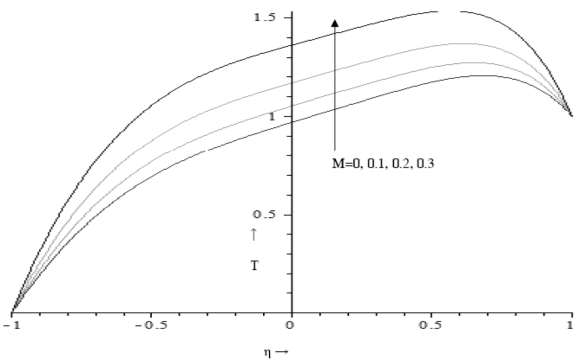


**Figure 4.** Temperature profile against the perpendicular distance from central line for different values of magnetic parameter M (when *EPr* = 1.0).

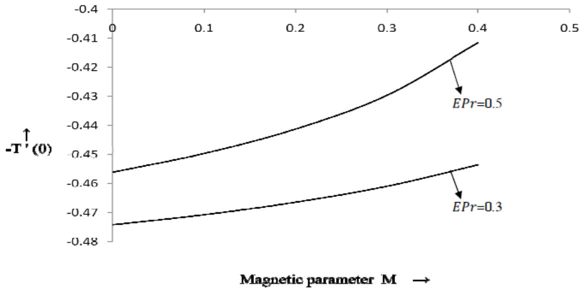


**Figure 5.** Coefficient of heat transfer, against magnetic parameter for different values of *EPr*.

## 5. Conclusions

A mathematical model has been presented for the problem of plane Poiseuille flow with unequal wall temperatures of an incompressible fluid having temperature dependent viscosity in the presence of transverse magnetic field. The governing boundary layer equations were solved numerically using the Runge-Kutta-Fehlberg method using Maple software. We infer from this study that the maximum velocity does not occur in the middle of the channel but moves towards the upper wall as the value of M and $\alpha$ increases. The temperature and $-T'(0)$ increases with the increasing value of M and *EPr*, this heat transfer is very important in production engineering to improve the quality of the final product.

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## Nomenclature

$B_0$ Constant applied magnetic field $(Wb_{m^{-1}})$

$C_p$ Specific heat at constant pressure $(J\,kg^{-1}K^{-1})$

$E$ Eckert number

$h$ Distance of the walls from the middle of the channel (*m*)

$M$ Dimensionless magnetic field parameter

$p$ Pressure (Pa)

$Pr$ Prandtl number

$\bar{T}$ Temperature (*K*)

$u$ Velocity component in x direction ($ms^{-1}$)

$u_m$ Maximum velocity ($ms^{-1}$)

$v$ Velocity component in y direction ($ms^{-1}$)

$x$ Dimensional horizontal coordinate (*m*)

$y$ Dimensional vertical coordinate (*m*)

**Greek symbols**

$\eta$ Dimensionless space variable

$\sigma_e$ Current density ($Cm^{-2}s^{-1}$)

$x$ Thermal conductivity ($m^2s^{-1}$)

$\mu$ Dynamic viscosity (Pa s)

**Superscript**

\- Dimensional quantities

**Subscript**

1 Condition at the upper wall

0 Condition at the lower wall

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