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A Model For The Residence Time Distribution and Holdup Measurement in a Two Impinging Streams Cyclone Reactor/Contactor in Solid-Liquid Systems

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Abstract
In this paper a two impinging streams cyclone contacting system suitable for handling of solid-liquid systems has been studied. Certain pertinent parameters such as: solid holdup, mean residence time and Residence Time Distribution (RTD) of solid particles have been investigated. A stochastic model based on Markov chains processes has been applied which describe the behavior of solid particles in the contacting system. From this model the RTD data were estimated and compared with the experimental results. The RTD data were obtained at different Δt *and compared with those estimated from the model. At* $\Delta t = 0.362$ s *a good correlation has been observed between the predicted and experimental data. The RTD data may be used to determine certain pertinent characteristic parameters of physical and chemical apparatuses such as conversion in chemical reactors.*

Keywords: *Holdup, Cyclone contactor, Impinging streams, Residence time distribution, Markov chains model*

Introduction

The application of two impinging streams systems as chemical reactors was first reported in 1982 by Elperin and Tamir [1].
In such a contacting system, two feed streams flowing either parallel or counter currently collide with each other in the impingement zone. These systems may be applied to physical and chemical processes such as drying [2, 3], gas-gas and solid-solid mixing [4,5], dissolu-tion of solids in solvents [6], extraction [7, 8, 9, 10], liquid-liquid reactions [11, 12] and solid-liquid reactions [13].
The two impinging streams cyclone reactor (TISCR) was first designed and applied by Tamir et al. in 1989 [14]. Oscillating of disperse phase particles in continuous phase is the main characteristic of cyclone contacting systems which leads to increase in residence time of disperse phase particles in the continuous phase.
The objective of this work was to evaluate some pertinent parameters affecting the performance of system behavior such as: mean residence time, residence time distribution of solid particles and the holdup of liquid and solid phases.

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Experimental system

The Apparatus

The vessel was a tangential horizontal flow of two impinging streams systems. The apparatus set up comprised a contacting chamber made of Pyrex glass, 43 cm height, 12 cm diameter and 5 mm thickness, with an inner pipe creating annular space and directing the streams toward the impinging zone and also allowing the gaseous products to leave the system. The diameter of the central inner tube was 22 mm. A schematic of the diagram of the vessel is shown in Figure 1.

Two identical nozzles made of stainless steel with an outlet port diameter of 2 mm and an inlet port diameter of 4.5 mm were placed at the entrance of two liquid lines. The distance between the nozzles and the apparatus entrance was 9 cm. Application of the nozzles leads to a drop in pressure and hence, increase in the outlet velocity. The latter permits the solids to enter the liquid stream, thus promoting the mixing intensity.

Two mixed streams entered the vessel via two silicon pipes having 6.5 mm inner diameter and 12 mm outlet diameter. Solids were fed to the liquid streams by means of two tapped glass vessels placed at 2 cm distance from the nozzles.

The liquid streams were pumped at high velocity, using a stainless steel centrifugal pump with a capacity of 50-400 L/h. A rotameter with a measuring capability of 60-600 L/h of liquid was used to measure the liquid flow rate.

Materials

Sand grains with mesh 50 and 2.5 g/cm^3 density were used as the solid phase while water was applied as the liquid phase. The experiments were performed at the ambient temperature (25-30 ºC).

Results and Discussion

To present a suitable model for the contacting system, the behavior of solid particles should be known in the latter.

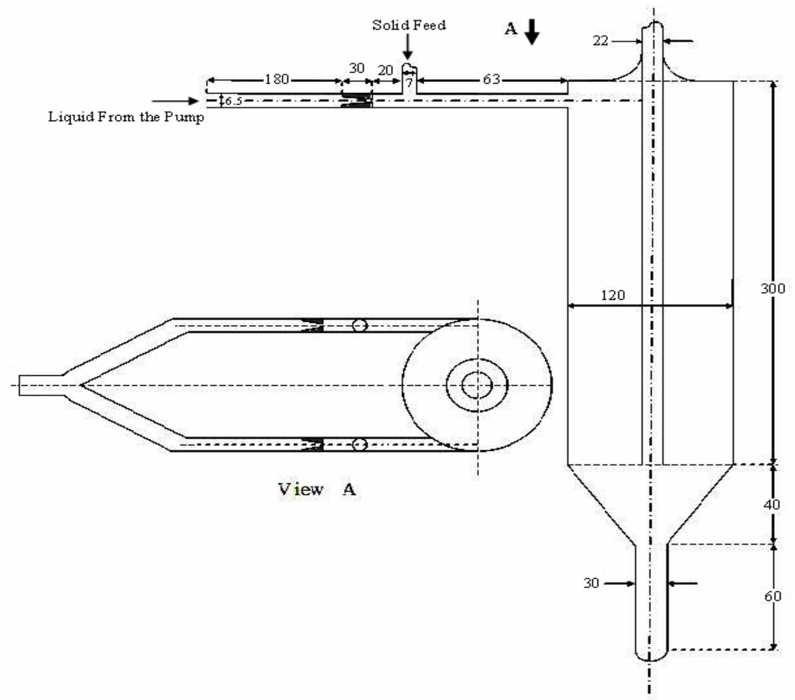

Figure 1. Schematic diagram of the contacting system and feeders. All dimensions are in millimetres.

Holdup and Mean residence time

To determine the mean residence time of particles within the system, the holdup of solid and liquid phases should be known. The holdup, υ, was measured by stopping the flow of both particles and water and collecting the materials remaining in the vessel. The liquid phase volume and the weight of sands were measured and reported as liquid and solid holdup. In Figs. 2 and 3, the solid holdup as a function of the liquid flow rates and the liquid holdup as a function of the solid flow rates are presented.

The general conclusions drawn from Fig. 2 may be summarized as follows:

(1) At a given liquid flow rate, increasing the solid mass flow rate increases the solid hold up.

(2) By increasing the liquid flow rate, the solid holdup is also increased.

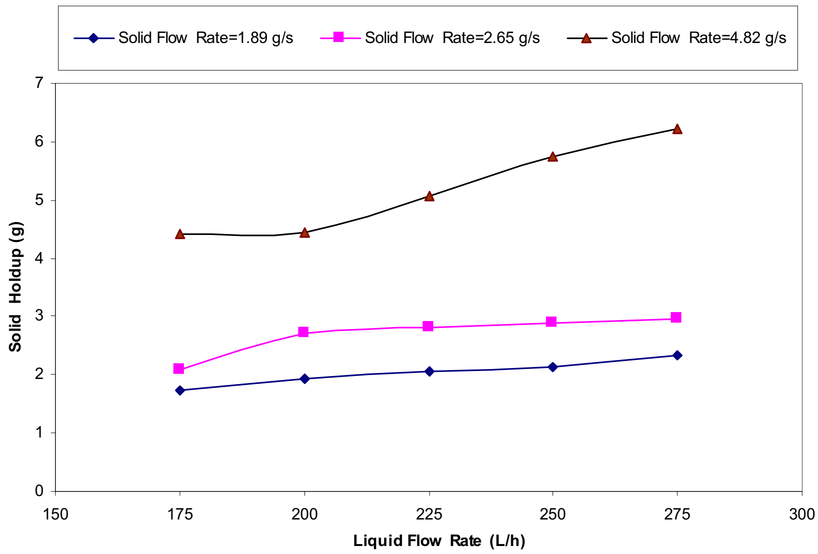

Figure 2. Holdup of solid particles within the contactor

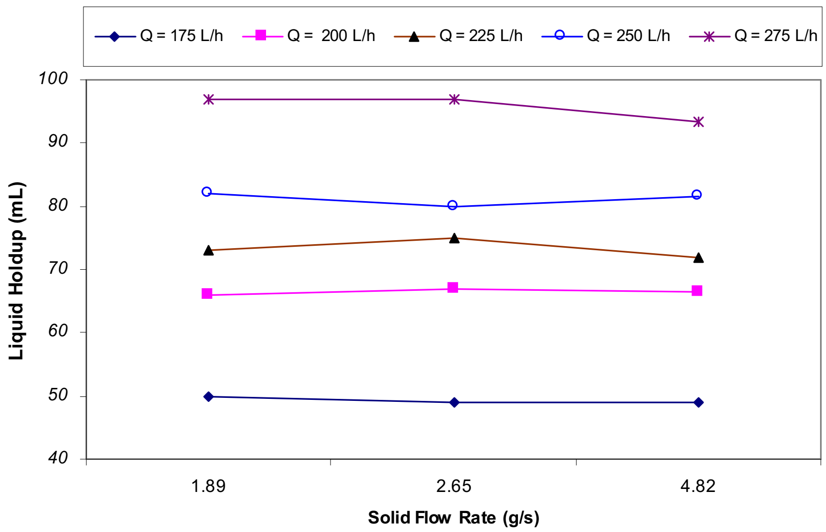

Figure 3. Liquid holdup as a function of solid mass flow rate

By considering the density and size of the particles applied in this study this behavior may be explained by the enhancement of the particle oscillation and rotation when the liquid flow rate is increased. It is apparent from Fig. 3 that at a given liquid flow rate, the hold up of liquid phase in the contacting system is almost constant and insensitive to the change in the solid flow rate. Such a behavior may be explained by noting that the flow rate of liquid in all the experimental runs was greater than that of the solid. Hence, variation in the solid flow rate may not have a profound effect on the liquid hold up.

In Fig. 4, it is demonstrated that in all solid flow rates, a slight increase in solid residence time occurrs as the liquid flow rate is increased.

RTD of solid particles

RTD experiments were conducted to obtain insight into the behavior of the particles in the contacting system. This is an important parameter because of the particles oscillatory motion that may produce internal recirculation and affect the mean residence time in the contacting system. The stimulus–response method was applied to determine the residence time distribution [15]. In order to perform the RTD experiments, a circular plate divided into 23 equal segments was constructed. The plate was placed under the solid discharge port of the contacting system and was driven at the selected speed by means of an electric motor. Different sampling intervals were thus obtained by changing the speed of rotation. A suitable tracer, having visible effects in the outlet stream should be used. In the present study sand grains with mesh 50 were applied as the tracer. About 4 *g* of tracer was rapidly injected into the inlet streams and the outlet stream from the contactor was conducted to the rotating disk. The experiments for RTD determination were performed at different disk rotation velocities. Plotting of the solid particles weight in each cell versus $n\Delta t$ gives the RTD curve of the contacting system. Experimental RTD charts at various Δt are shown in Figs. 4 to 7. It is apparent from Fig. 4 that in small Δt values, the RTD curves have demonstrated certain degree of random fluctuations. Increase in Δt, lowers the sensitivity of reading until at a certain value of Δt, random fluctuations of particles are no longer observed. The experimental results were used to calculate the mean residence time and the variance of RTD data according to equations 1 and 2, respectively.

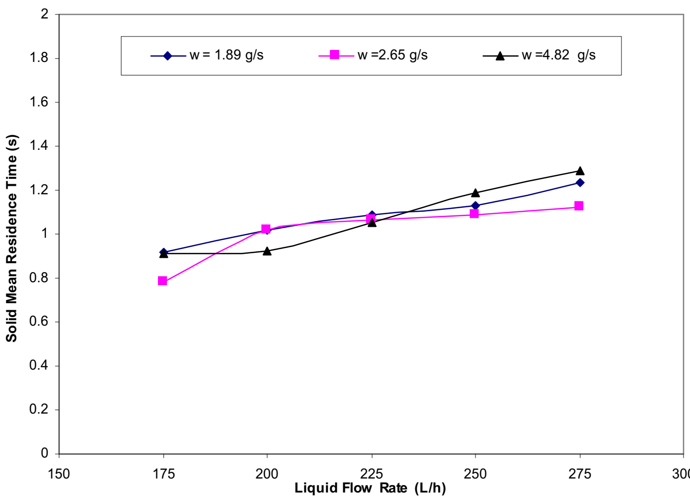

Figure 4. Variation of the mean residence time of solid particles within the contactor

$$\bar{t}_{\exp} = \frac{\sum_{i=1}^{23} t_i m_i \Delta t_i}{\sum_{i=1}^{23} m_i \Delta t_i} \qquad (1)$$

$$\sigma_{\exp}^{2} = \frac{\sum_{i=1}^{23} (t_i - \bar{t}_{\exp})^2 m_i \Delta t_i}{\sum_{i=1}^{23} m_i \Delta t_i} \qquad (2)$$

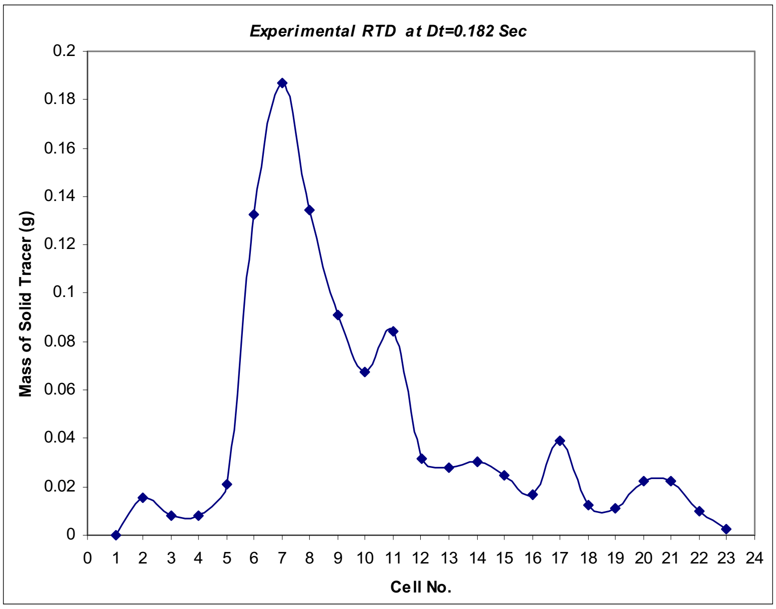

Figure 5. Experimental RTD at $\Delta t = 0.182$ s

$\sigma_{\exp}^2 = 1.516\ s^2$, $\bar{t}_{\exp} = 1.798$ s, $Q = 200\ L/h$

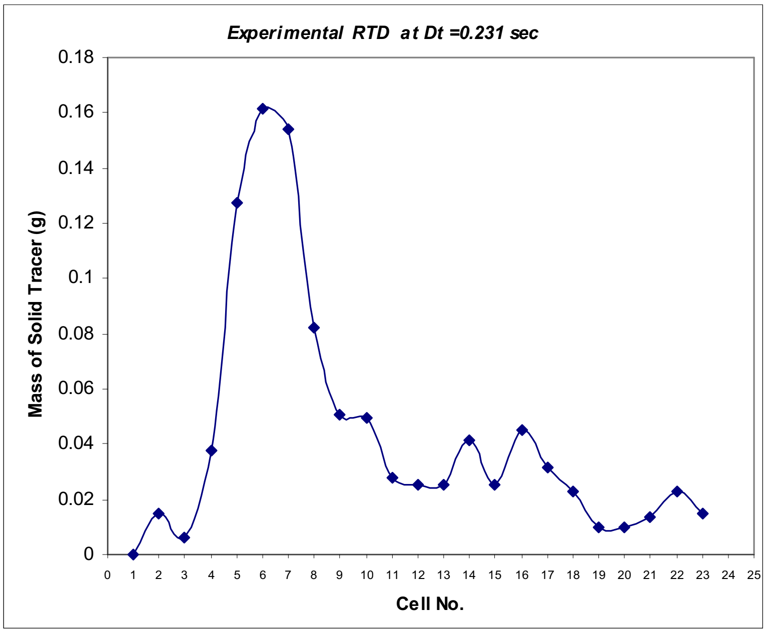

Figure 6. Experimental RTD at $\Delta t = 0.231$ s

$\sigma_{\exp}^2 = 1.366\ s^2$, $\bar{t}_{\exp} = 2.213$ s, $Q = 200\ L/h$

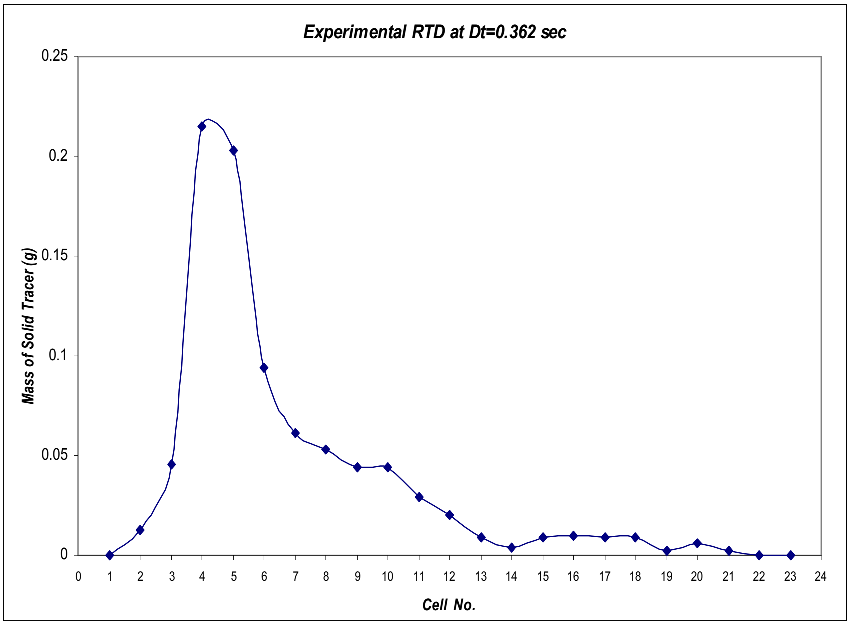

Figure 7. Experimental RTD at $\Delta t = 0.362$ s

$\sigma^2_{exp} = 1.516\ s^2$, $\bar{t}_{exp} = 2.307$ s, $Q = 200\ L/h$

Markov chains model

As the collision of particles in the impingement zone is random, a suitable mathematical technique to handle such a process could be Markov chains models [16]. According to discrete-time Markov chains, the probability of an event at time *t* given only the outcome at time *t-1* is equal to the probability of the event at time *t* given the entire history of the system. In other words, the probability of the event at *t* is not dependent upon the state history prior to time *t-1*. Thus, the values of the process at the given time *t-1* determine the conditional properties for future values of the process. These values are called the state of the process, and the conditional properties are thought of as transition probabilities between the states *i* and *j*, p_{ij}. These values may be displayed in a matrix $(P = \lfloor p_{ij} \rfloor)$ called the transition matrix.

The matrix **P** has *N* rows and *N* columns, where *N* is the number of possible states for transition of the system.

The rows of matrix **P** consist of the probabilities of all possible transitions from a given state and, as such, sum to 1.

$$\sum_{j=1}^{N} p_{ij} = 1 \qquad (3)$$

This matrix completely describes the Markov process. Other definitions and equations related to Markov processes are as follows:

(I) $s_i(n)$, the state probability, defined as the probability that the system will be in the state *i* after n transitions from a given starting point.

(II) $S(n)$, the state probability vector, a line vector composed of elements $s_i(n)$. $S(n) = [s_1(n) \quad s_2(n) \quad \ldots \quad s_N(n)]$. The equations which govern the Markov processes are as follows:

$$\sum_{m=1}^{N} s_i(n) = 1 \qquad (4)$$

This equation implies that the total probability of the system to reach on of all possible *N* states which can be occupied after *n* transitions is equal to unity. Equation 5 states that the probability that the system is in state *j* after *n+1* transition is equal to the sum of its probabilities of being in any state *i* after n transitions, multiplied by the probability of transferring from state *i* to state *j* in one transition step.

$$s_j(n+1) = \sum_{i=1}^{N} s_i(n) p_{ij} \quad n = 0, 1, 2, \ldots \tag{5}$$

Further details of Markov models may be found elsewhere [17, 18, 19].

By considering the patterns of particle flow within the vessel, the reaction compartment was divided into seven regions with equal volumes (Fig. 8). Each region was assumed to be an ideal flow contactor. A recycle stream *r* was also considered due to counter-current flows in the impingement zone. Each region represents a state in the Markov process, and the transition probability p_{ij} is the probability of a solid particle leaving region *i* and entering region *j*.

Now, consider region 7 shown in Fig. 8. Whenever a particle reaches this region, it leaves the system to state 8 where it remains. State 8 is then a *trapping state*.

The seventh element of the state probability vector $s_7(n)$ indicates the probability that a particle entering the system at $n=0$ will leave the system after $n\Delta t$ $(n=1,2,3,...)$. The numerical value of $s_7(n)$ for $n=0, 1, 2, \ldots$, represents the impulse response of the system after *n* intervals of time length Δt.

It is possible to define the various probabilities along the path of a particle from the inlet to the system down to the exit due to the assumption of ideal flow regions for each state. For ideal plug regions, $p_{ii}=0$ and for ideal mixed regions, $p_{ii} = \exp(-\Delta t/\tau)$, while the probability of leaving any vessel *i* is equal to $1-p_{ii}$ [16, 17]. Therefore,

$$p_{11} = p_{22} = 0$$

$$p_{33} = p_{44} = e^{\left(-\frac{\Delta t}{\tau}\right)} = e^{(\alpha)} = \exp\left(-\frac{\Delta t}{\upsilon}\left(\frac{w_p}{2}+\frac{r}{2}\right)\right)$$

$$= \exp\left(-n_v \frac{\Delta t}{\bar{t}}\left(\frac{1}{2}+\frac{r}{2w_p}\right)\right) = \exp\left(-n_v \Delta\theta \frac{R+1}{2}\right) = a$$

$$p_{35} = p_{45} = \frac{R+0.5}{R+1}\left(1-p_{33}\right) = b \tag{6}$$

$$p_{36} = p_{46} = \frac{0.5}{R+1}\left(1-p_{33}\right) = c$$

$$p_{55} = e^{\beta} = d$$

$$p_{56} = \frac{1}{1+2R}\left(1-p_{55}\right) = e$$

$$p_{53} = p_{54} = \frac{R}{R+1}\left(1-p_{55}\right) = f$$

In which:

$$\beta = -n_v \Delta\theta (R+0.5)$$

$$\alpha = -n_v \Delta\theta \left(\frac{R+1}{2}\right) \tag{7}$$

Other components of the transition matrix are equal to zero. In addition, the complete transition probabilities matrix may be constructed as follows:

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & b & c & 0 & 0 \\ 0 & 0 & 0 & a & b & c & 0 & 0 \\ 0 & 0 & f & f & d & e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

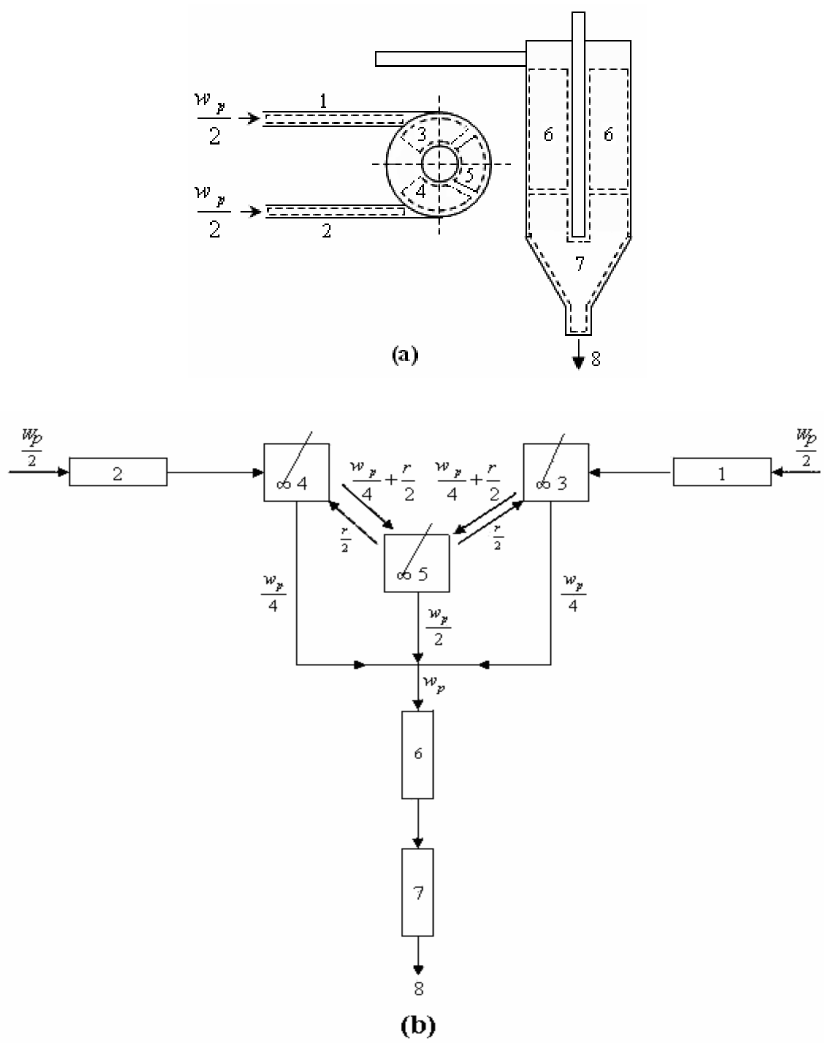

Figure 8. Modeling of the contacting system: (a) flow regions proposed for the contacting system; (b) the resulting model. 1, 2, 6, 7: perfect plug regions. 3, 4, 5: perfect mixed regions

The initial state probability vector $s_N(0)$ is also known:

$$\mathbf{s}(0) = [0.5, 0.5, 0, 0, 0, 0, 0, 0] \tag{9}$$

In order to determine the RTD it is essential to calculate $s_7(m)$ elements for all *m* values from 1 to 23 and plot the results versus time$(n\Delta t)$. According to the RTD definition the following equation is valid:

$$RTD = \sum_{m=1}^{23} s_7(m) = f(\Delta\theta, R) \tag{10}$$

The variance of the theoretical RTD curve was determined from the following equation [17]:

$$\sigma_{\text{mod}}^2 = \sum_{m=1}^{23} s_7(m)\left[\bar{t}(m\Delta\theta - 1)\right]^2 \tag{11}$$

By considering the implication of the equality of the two mean residence times of the experimental and theoretical RTD curves, the following relation is obtained [17]:

$$\Delta\theta \sum_{m=1}^{23} m\, s_7(m) = 1 \tag{12}$$

In this equation, $s_7(m)$, which is initially a function of both R and $\Delta\theta$, become solely a function of R after replacing $\Delta\theta$ values.
A computer program was developed in *C++* code to calculate $s_7(m)$ elements. The flow chart of the program is presented in Fig. 9.
A comparison has been made between the experimental data and those predicted from the model. This is presented in Table 1.

Table 1. Dependency of $\bar{\mathbf{t}}_{exp}(\mathbf{s})$, $\boldsymbol{\sigma}^2_{exp}(\mathbf{s}^2)$ and $\boldsymbol{\sigma}^2_{mod}(\mathbf{s}^2)$ on $\boldsymbol{\Delta t}$

$\Delta t(s)$	$\sigma^2_{exp}(s^2)$	$\bar{t}_{\exp}(s)$	$\sigma^2_{mod}(s^2)$	R
0.182	0.635	1.798	1.049	0.601
0.231	1.366	2.213	1.637	0.314
0.362	1.516	2.307	1.525	0.214

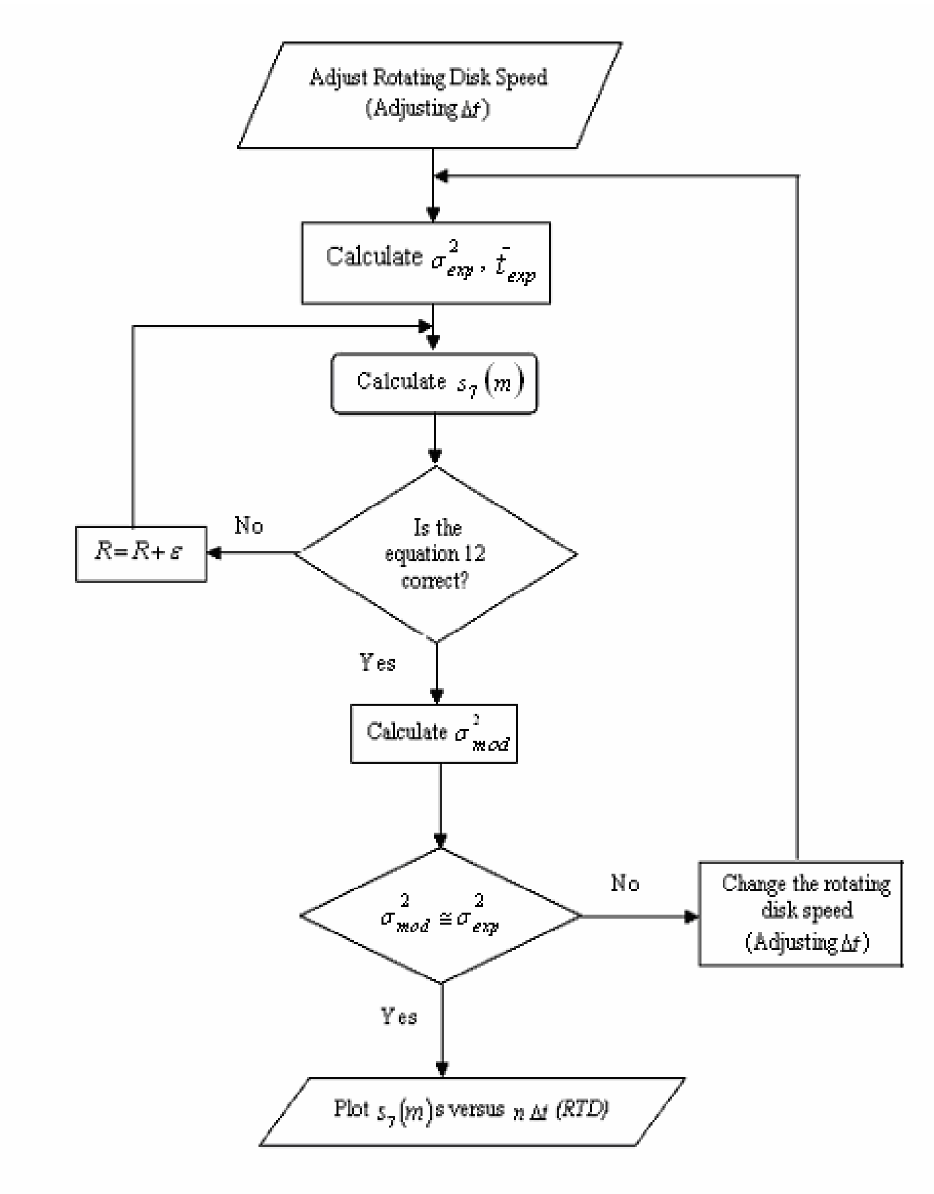

Figure 9. Computer program flowchart

It may be observed from this table that with Δt equal to *0.362 s* a good correlation is obtained between the experimental data and model predictions. This is demonstrated in Fig. 10. This is achieved by comparing the variances of the experimental and predicted data at each Δt. The minimum difference between σ^2_{mod} and σ^2_{exp} may relate to the best correlation between the two sets of RTD data.

It may be observed from Fig. 10 that about 21.4% of the total solid flow (w_p) oscillate between regions 3, 4 and 5.

Conclusion

Certain pertinent parameters affecting the performance capability of the tangential flow of two impinging streams cyclone contactors with recycling such as mean residence time, liquid and solid holdups and residence time distribution of solid particles were studied.

Increasing the liquid flow rate (up to 300 L/h) increases the solid mean residence time within the system. This could in turn enhance the conversion of reactants in the impinging streams chemical contacting systems.

A compartment model was proposed to describe the flow pattern within the contacting system. On the basis of such a flow pattern and application of Markov chains discrete formulation, a theoretical two parameter model was derived for the RTD of the solid phase. The two parameters were evaluated, and the model was adjusted using experimental RTD data. The model developed in the present study, may be applied to estimate the RTD for other vessels having a geometry similar to that used in this investigation. To determine the RTD of particles within any other impinging contactor, a suitable flow region should be proposed for that vessel and the predicted results from the present model have to be compared with the experimental data available for that particular apparatus.

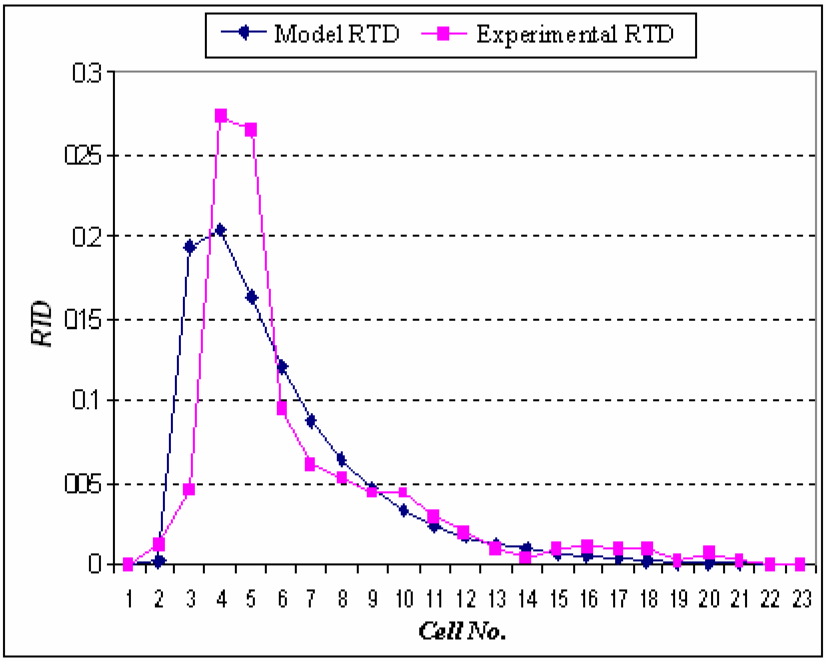

Figure 10. Comparison between the experimental and theoretical RTDs in the contacting system. Each increment in the *x* axis represents a $\Delta t = 0.362$ s

$$\bar{t}_{exp} = 2.307\,s,\ \sigma^2_{exp} = 1.516\ s^2,\ \sigma^2_{mod} = 1.525\ s^2,\ R = 0.214,\ Q = 200\ L/h\ \text{and}\ w_p = 0.616\ g/s.$$

Notations

a = constant eq. 6
b = constant eq. 6
c = constant eq. 6
d = constant eq. 6
e = constant eq. 6
f = constant eq. 6
m_i = mass of solid particle in each cell
N = number of all transition probability
n = cell indicator
n_υ = number of regions in the flow model
p_{ij} = transition probability
$\boldsymbol{P}$ = transition matrix
Q = liquid flow rate (dm^3/h)
r = solid recycle flow rate (g/s)
R = solid recycle ratio = r/w_p
RTD = residence time distribution
$s_i(m)$ = state probability
$\mathbf{S}(m)$ = state probability vector
$s_i(m)$ = ith element of the $\mathbf{S}(m)$ vector
t = time (s)
$\bar{t}$ = mean residence time (s)
w_p = solid mass flow rate (g/s)
α = constant eq. 10
β = constant eq. 10
Δt = time increment (s)
$\Delta\theta$ = dimensionless time = $\Delta t/\bar{t}$
σ^2 = variance (s^2)
υ = solid holdup (g)

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