

## Mathematical modeling of a fixed bed chromatographic reactor for Fischer Tropsch synthesis

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### Abstract

*In this research, Fischer Tropsch synthesis (FTS) has been modeled in the fixed bed chromatographic reactor for the first time by applying a rather complex dispersed plug flow model for fluid phase and linear driving force (LDF) model for adsorbent. Model equations are dynamic, multi-component, non-linear and heterogeneous including reaction and adsorption simultaneously. Complex kinetics for FTS and water-gas shift (WGS) reaction and the multicomponent Langmuir adsorption isotherm is used in the model. A set of partial differential and ordinary differential equations with algebraic equations have been converted into a set of ordinary differential equations by using the orthogonal collocation technique. Then this set of equations has been solved by multi-step methods of Numerical Differentiation Formulae (NDF) or Backward Differentiation Formulae (BDF) known as the Gear's method. Consequently, results for dynamic model and effects of modeling parameters have been analyzed. Through this fixed bed chromatographic reactor model, one may develop a suitable configuration of simulated moving bed chromatographic reactors.*

**Keywords:** *Dynamic Modeling, Fixed Bed Chromatographic Reactors, Adsorption, Fisher Tropsch Synthesis and Orthogonal Collocation*

### Introduction

Chromatographic reactors perform chromatographic separation operations based upon adsorption and catalytic reactions by solid particle catalysts in a reactor column, containing a blend of catalyst and adsorber, simultaneously. Reactants and carrier streams are fed into the reactor from the inlet as a fluid phase. Reactions occur during movement of these materials along the reactor column and parts of them are converted into products. The success of these reactors depends on the selective adsorption.

Reactants and products' adsorption must be varied enough to achieve a good separation. FTS has been modeled in fixed bed and slurry bubble column reactors in several works such as [1-6] but it has not been modeled in chromatographic reactors. Also there are some references including modeling of chromatographic reactors and separators in fixed bed and simulated moving bed such as [7-14], but they did not use FTS as their reactions. Because there is no other article pertaining to modeling of FTS in chromatographic reactors, in this work, FTS has been

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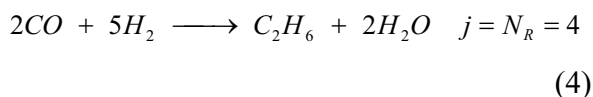
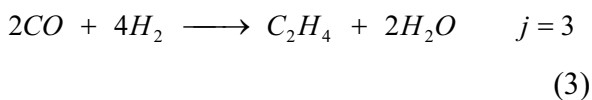
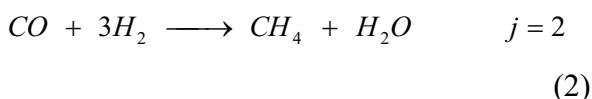
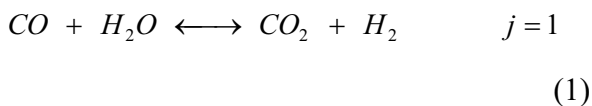
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modeled in such reactors for the first time. However, the model was developed under isothermal and unsteady state conditions.

There is a thorough review of these reactors and modeling of them in reference [15]. Here, the FTS and WGS reactions are modeled simultaneously in a chromatographic reactor. Many researchers have investigated modeling of chromatographic separations and different theories with different complexities are presented [16-18]. These theories are based upon adsorption models and discussed in detail elsewhere [16]. A comprehensive review on dynamic and modeling of adsorption and isothermal chromatography is presented by Ruthven [17] where he has divided such models into three classes [17, 18]: equilibrium theory, plate model and rate model. In this research, the rate model with a linear driving force rate expression is used.

### Process for modeling

In this section the process has been used for modeling is discussed: the reactor column is taken to be a cylinder with length L and inner diameter D that is packed with catalyst and adsorber. The adsorber selected is Zeolite-5A and the catalyst is determined to be Fe-Cu-K. The feed mixture contains CO, H<sub>2</sub> and carrier gas (He). The feed is passed through the chromatographic reactor. During the reaction, separation through adsorption also occurs. The main reactions are a combination of FTS up to C<sub>2</sub> and WGS:



$$[j] = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \equiv \begin{bmatrix} WGS(1):CO_2 \text{ production} \\ FTS1(2):CH_4 \text{ production} \\ FTS2(3):C_2H_4 \text{ production} \\ FTS3(4):C_2H_6 \text{ production} \end{bmatrix} \quad (5)$$

$$\alpha = [\alpha_{ij}] = \begin{bmatrix} -1 & -1 & -2 & -2 \\ 1 & -3 & -4 & -5 \\ 1 & 0 & 0 & 0 \\ -1 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} CO(i=1) \\ H_2(i=2) \\ CO_2(i=3) \\ H_2O(i=4) \\ CH_4(i=5) \\ C_2H_4(i=6) \\ C_2H_6(i=7) \\ He(i=8) \end{matrix} \quad (6)$$

$$\varepsilon_a = \frac{\text{Adsorber Volume}}{\text{Total Bed Volume}} \quad (7a)$$

$$\varepsilon_b = \frac{\text{Fluid Volume}}{\text{Total Bed Volume}} \quad (7b)$$

$$\varepsilon_c = \frac{\text{Catalyst Volume}}{\text{Total Bed Volume}} \quad (7c)$$

Where j is the reaction path number from 1 to N<sub>R</sub> and N<sub>R</sub> is the total number of reactions.

α<sub>ij</sub> is the stoichiometric coefficient of component i in the reaction path j and volume fraction of adsorber, the fluid and catalyst are ε<sub>a</sub>, ε<sub>b</sub> and ε<sub>c</sub>, respectively.

### Model Assumptions

The main assumptions utilized in this research include:

- 1- Plug flow with axial dispersion for fluid flow model

- 2- Isothermal conditions
- 3- Linear driving force (LDF) approach is adapted for the adsorber
- 4- Pseudo-homogeneous model for catalytic reactions
- 5- Fluid velocity is taken to be constant with an average value
- 6- Pressure drop of the packed bed is taken to be negligible because of low velocity and
- 7- Langmuir adsorption isotherm is used for multicomponent adsorption equilibrium.

It is noteworthy that, this model is dynamic as it employs the axial dispersion model with linear driving force.

### Model Equations

There are two classes of differential equations based upon the above assumptions [15]:

- 1- Partial mass balance for every component  $i$  in the fluid moving through the bed and
- 2- Linear driving force equation for every component  $i$  in the adsorber beads

Moreover, there are two classes of algebraic equations including:

- 1- Reaction kinetics rates and
- 2- Adsorption equilibrium isotherm equations

All are incorporated into the present model.

Ultimately, developed equations for this model are as follows:

### I - Fluid phase (PDEs):

$$\frac{\partial C_{bi}}{\partial t} = -v \frac{\partial C_{bi}}{\partial Z} + D_{Li} \frac{\partial^2 C_{bi}}{\partial Z^2} + \frac{\varepsilon_c}{\varepsilon_b} \rho^{cat} \times \sum_{j=1}^{N_R} \alpha_{ij} R_j - \frac{\varepsilon_a}{\varepsilon_b} k_{Ci} \left( \frac{b_i C_{psoi} C_{bi}}{1 + \sum_{j=1}^{N_C} b_j C_{bj}} - C_{pi} \right) \quad (8)$$

### II - Adsorbent phase (ODEs):

$$\frac{\partial C_{pi}}{\partial t} = k_{Ci} \left( \frac{b_i C_{psoi} C_{bi}}{1 + \sum_{j=1}^{N_C} b_j C_{bj}} - C_{pi} \right) \quad (9)$$

### III - Related initial and boundary conditions

I.C.1:

$$t = 0 \quad ; \quad C_{bi} = C_{bi0} = C_{bi}(0, Z) \quad ; \quad 0 < Z \leq L \quad (10)$$

I.C.2:

$$t = 0 \quad ; \quad C_{pi} = C_{pi0} = C_{pi}(0, Z) \quad ; \quad 0 \leq Z \leq L \quad (11)$$

B.C.1:

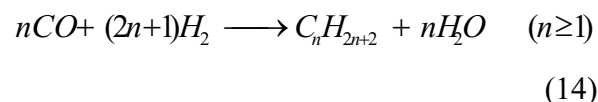
$$Z = 0 \quad ; \quad \frac{\partial C_{bi}}{\partial Z} = \frac{v}{D_{Li}} (C_{bi} - C_{bi,input}) \quad ; \quad t > 0 \quad (12)$$

B.C.2:

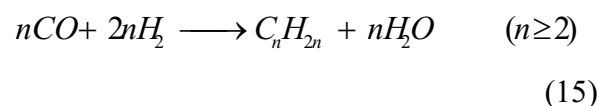
$$Z = L \quad ; \quad \frac{\partial C_{bi}}{\partial Z} = 0 \quad ; \quad t > 0 \quad (13)$$

In addition, reactions may be summarized in the following parts:

1- Paraffin formation reactions:



2- Olefin formation reactions:



3- Water gas shift reaction:



**Table 1.** Stoichiometric coefficients matrix for FTS and WGS reactions

Reactants	Reaction Path					
	CO+H <sub>2</sub> O	CO+3H <sub>2</sub>	2CO+4H <sub>2</sub>	2CO+5H <sub>2</sub>	nCO+2nH <sub>2</sub>	nCO+(2n+1)H <sub>2</sub>
Products	CO <sub>2</sub> +H <sub>2</sub>	CH <sub>4</sub> + H <sub>2</sub> O	C <sub>2</sub> H <sub>4</sub> + 2H <sub>2</sub> O	C <sub>2</sub> H <sub>6</sub> + 2H <sub>2</sub> O	C <sub>n</sub> H <sub>2n</sub> +n H <sub>2</sub> O	C <sub>n</sub> H <sub>2n+2</sub> +n H <sub>2</sub> O
CO	-1	-1	-2	-2	-n	-n
H <sub>2</sub>	1	-3	-4	-5	-2n	-(2n+1)
CO <sub>2</sub>	1	0	0	0	0	0
H <sub>2</sub> O	-1	1	2	2	n	n
CH <sub>4</sub>	0	1	0	0	0	0
C <sub>2</sub> H <sub>4</sub>	0	0	1	0	0	0
C <sub>2</sub> H <sub>6</sub>	0	0	0	1	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
C <sub>n</sub> H <sub>2n</sub>	0	0	0	0	1	0
C <sub>n</sub> H <sub>2n+2</sub>	0	0	0	0	0	1

FTS and WGS Reaction Rates for Fe-Cu-K catalyst used based upon equations (17)-(20) [19] are presented in the following:

$$R_{CH_4} = \frac{k_{5M} P_{H_2} \alpha_1}{1 + \left(1 + \frac{1}{K_2 K_3 K_4} \frac{P_{H_2O}}{P_{H_2}^2} + \frac{1}{K_3 K_4} \frac{1}{P_{H_2}} + \frac{1}{K_4}\right) \sum_{i=1}^N \left(\prod_{j=1}^i \alpha_j\right)} \quad (n = 1) \quad (17)$$

$$R_{C_n H_{2n+2}} = \frac{k_5 P_{H_2} \prod_{j=1}^n \alpha_j}{1 + \left(1 + \frac{1}{K_2 K_3 K_4} \frac{P_{H_2O}}{P_{H_2}^2} + \frac{1}{K_3 K_4} \frac{1}{P_{H_2}} + \frac{1}{K_4}\right) \sum_{i=1}^N \left(\prod_{j=1}^i \alpha_j\right)} \quad (n \geq 2) \quad (18)$$

$$R_{C_n H_{2n}} = \frac{k_6 (1 - \beta_n) \prod_{j=1}^n \alpha_j}{1 + \left(1 + \frac{1}{K_2 K_3 K_4} \frac{P_{H_2O}}{P_{H_2}^2} + \frac{1}{K_3 K_4} \frac{1}{P_{H_2}} + \frac{1}{K_4}\right) \sum_{i=1}^N \left(\prod_{j=1}^i \alpha_j\right)} \quad (n \geq 2) \quad (19)$$

$$R_{CO_2} = \frac{k_V (P_{CO} P_{H_2O} / P_{H_2}^{0.5} - P_{CO_2} P_{H_2}^{0.5} / K_P)}{1 + K_V P_{CO} P_{H_2O} / P_{H_2}^{0.5}} \quad (20)$$

Partial pressure of component i is calculated based upon the equation (21):

$$P_i = \left( \frac{m_i}{\sum_{i=1}^{N_c} m_i} \right) P_t \quad (21)$$

$P_t$  is the total pressure of the reactor and  $m_i$  is the molar flow rate of component i.  $\alpha_1$  and  $\alpha_n$  are calculated from equations (22)-(23):

$$\alpha_1 = \frac{k_1 P_{CO}}{k_1 P_{CO} + k_{5M} P_{H_2}} \quad (n=1) \quad (22)$$

$$\alpha_n = \frac{k_1 P_{CO}}{k_1 P_{CO} + k_5 P_{H_2} + k_6 (1 - \beta_n)} \quad (n \geq 2) \quad (23)$$

$\beta_n$  in equation (23) is defined in the following form:

$$\beta_n = \frac{k_{-6} P_{C_n H_{2n}}}{k_6 \alpha_A^{n-1} \frac{k_1 P_{CO}}{k_1 P_{CO} + k_5 P_{H_2}} + \frac{k_{-6}}{k_1 P_{CO} + k_5 P_{H_2} + k_6} \sum_{i=2}^n (\alpha_A^{i-2} P_{C_{n-i+2} H_{2(n-i+2)}})} \quad (24)$$

$(n \geq 2)$

$\alpha_A$  in equation (24) is defined in equation (25):

$$\alpha_A = \frac{k_1 P_{CO}}{k_1 P_{CO} + k_5 P_{H_2} + k_6} \quad (25)$$

Reaction rate constants are changed with temperature according to the Arrhenius equation:

$$k_i(T) = k_{i,0} \exp\left(\frac{-E_i}{RT}\right) \quad (26)$$

Optimum values for reaction rate parameters are summarized in table 2 [19]:

Equilibrium constant for the WGS reaction is estimated based upon equation (2<sup>v</sup>) as a function of temperature [19]:

$$\ln(K_p) = \frac{5078.0045}{T} - 5.8972089 + (13.958689 \times 10^{-4})T - (27.592844 \times 10^{-8})T^2 \quad (27)$$

Multicomponent adsorption parameters of the Langmuir isotherm are given from reference [20]. The Langmuir isotherm is implemented as:

$$\frac{q_i^{*eq}}{q_{\infty i}} = \frac{b'_i P_i}{1 + \sum_{j=1}^{N_c} b'_j P_j} \quad (28)$$

**Table 2.** FTS and WGS reactions rates parameters [19]

Parameter	Dimension	Values	t-student dispersion
$k_1$	mole gr <sup>-1</sup> s <sup>-1</sup> bar <sup>-1</sup>	$(2.23+0.28) \times 10^{-5}$	15.32
$k_{5M,0}$	mole gr <sup>-1</sup> s <sup>-1</sup> bar <sup>-1</sup>	$(4.65+0.29) \times 10^3$	30.50
$E_{5M}$	kJ mole <sup>-1</sup>	$(9.289+0.094) \times 10^1$	198.35
$k_{5,0}$	mole gr <sup>-1</sup> s <sup>-1</sup> bar <sup>-1</sup>	$(2.74+0.12) \times 10^2$	44.26
$E_5$	kJ mole <sup>-1</sup>	$(8.701+0.083) \times 10^1$	206.27
$k_{6,0}$	mole gr <sup>-1</sup> s <sup>-1</sup>	$(2.23+0.28) \times 10^{-5}$	8.87
$E_6$	kJ mole <sup>-1</sup>	$(1.1104+0.0213) \times 10^2$	102.01
$k_{v,0}$	mole gr <sup>-1</sup> s <sup>-1</sup> bar <sup>-1.5</sup>	$(1.57+0.022) \times 10^1$	142.32
$E_v$	kJ mole <sup>-1</sup>	$(4.508+0.149) \times 10^1$	59.37
$k_{-6}$	mole gr <sup>-1</sup> s <sup>-1</sup> bar <sup>-1</sup>	$(2.75+0.25) \times 10^{-5}$	21.50
$K_V$	bar <sup>-0.5</sup>	$(1.13+0.08) \times 10^{-3}$	28.28
$K_2$	-	$(1.81+0.16) \times 10^{-2}$	21.89
$K_3$	-	$(4.68+0.66) \times 10^{-2}$	13.89
$K_4$	-	$(2.26+1.02) \times 10^{-1}$	4.35

### Dimensionless Equations

Dimensionless variables and parameters are defined in the following manner:

a. dimensionless independent variables:

$$\bar{t} = \frac{ut}{L\varepsilon_b} = \frac{vt}{L} \quad (29)$$

$$\bar{Z} = \frac{Z}{L} \quad (30)$$

b. dimensionless dependent variables:

$$\bar{Y}_{bi} = \frac{C_{bi}}{C_{br}} \quad (31)$$

$$\bar{Y}_{pi} = \frac{C_{pi}}{C_{pr}} \quad (32)$$

c. dimensionless parameters:

$$\alpha = -1 \quad (33)$$

$$\beta_i = \frac{D_{Li}}{Lv} = \frac{\varepsilon_b D_{Li}}{Lu} \frac{1}{Pe_{Li}} \quad (34)$$

$$\gamma_i(\bar{Y}_{pj}, \bar{Y}_{bj}, s) = \frac{\varepsilon_c \rho^{cat}}{\varepsilon_b v C_{br}} \times \sum_{j=1}^{N_R} \alpha_{ij} R_j - \frac{\varepsilon_a C_{pr}}{\varepsilon_b C_{br}} \times \lambda_i(\bar{Y}_{pj}, \bar{Y}_{bj}, s) \quad (35)$$

$$\lambda_i(\bar{Y}_{pj}, \bar{Y}_{bj} s) = \frac{k_{Ci} L}{C_{pr} V} \left( \frac{b_i C_{poci} \bar{Y}_{bi} C_{br}}{1 + C_{br} \sum_{j=1}^{N_c} b_j \bar{Y}_{bj}} - \bar{Y}_{pi} C_{pr} \right) \quad (36)$$

$$\bar{Y}_{bi0} = \frac{C_{bi0}}{C_{br}} \quad (37)$$

$$\bar{Y}_{pi0} = \frac{C_{pi0}}{C_{pr}} \quad (38)$$

$$\bar{Y}_{bi,input} = \frac{C_{bi,input}}{C_{br}} \quad (39)$$

Dimensionless equations are summarized in the following:

*PDEs :*

$$\left( \frac{\partial \bar{Y}_{bi}}{\partial t} \right) = \alpha \left( \frac{\partial \bar{Y}_{bi}}{\partial Z} \right) + \beta_i \left( \frac{\partial^2 \bar{Y}_{bi}}{\partial Z^2} \right) + \gamma_i(\bar{Y}_{pj}, \bar{Y}_{bj} s) \quad (40)$$

$$ODEs: \quad \frac{\partial \bar{Y}_{pi}}{\partial t} = \lambda_i(\bar{Y}_{pj}, \bar{Y}_{bj} s) \quad (41)$$

$$I.C.1: \quad \bar{t} = 0 \quad ; \quad \bar{Y}_{bi} = \bar{Y}_{bi0} \quad (42)$$

$$I.C.2: \quad \bar{t} = 0 \quad ; \quad \bar{Y}_{pi} = \bar{Y}_{pi0} \quad (43)$$

$$B.C.1: \quad \bar{Z} = 0 \quad ; \quad \frac{\partial \bar{Y}_{bi}}{\partial \bar{Z}} = \frac{1}{\beta_i} (\bar{Y}_{bi} - \bar{Y}_{bi,input}) \quad (44)$$

$$B.C.2: \quad \bar{Z} = 1 \quad ; \quad \frac{\partial \bar{Y}_{bi}}{\partial \bar{Z}} = 0 \quad (45)$$

*i* is a component number index between 1 and  $N_c$  which is equal to 8.

### Solution by Orthogonal Collocation Method [15, 21-28]

Roots of Jacobean polynomial are applied for collocation points. Lagrange interpolation is used. Based on the orthogonal collocation method, position derivatives are formulated from the interpolated polynomial and based on the method of lines these are substituted into the partial differential equations (PDEs) to convert them to the ordinary differential equations (ODEs). Boundary conditions are applied in these equations. Algebraic equations of adsorber equilibriums and reactions rates are introduced into this equation. Two dimensional matrixes of variables are converted to a vector of variables by a one by one correspondence equation. A set of ODEs has been solved by multistep methods of numerical differentiation formulae (NDF) or backward differentiation formulae (BDF), known as the Gear's method. ODEs solving is down by MAT LAB [29] software.

### Results and Discussions

There are FTS chromatographic experimental results, only in Pakseresht's work [30]. Thus, results of this research are compared with those of Pakseresht's in Fig. 1. Related trends are in good agreement but there are differences in absolute values between the present model predictions and his experimental data. This might be due to a different kinetic rates of the catalyst utilized in Pakseresht's work [30] which is different to the Wang et al. work [19]. Furthermore, Pakseresht's catalysts are not experimentally tested for determining rate expressions or fitting kinetic parameters.

Some other model parameters are summarized in table 3. The model is solved by orthogonal collocation and the results of this run are indicated in Fig. 2a-e as an example.

In Fig. 2a, dimensionless concentrations of CO, H<sub>2</sub>, CO<sub>2</sub> & H<sub>2</sub>O in the chromatographic reactor bed vs. the dimensionless length of the reactor and dimensionless time are indicated. CO and H<sub>2</sub> are consumed along the

chromatographic reactor. Therefore, the concentrations of them are reduced while  $z$  is increased from zero to one. The chromatographic reactor bed is full of He at start up, so CO and H<sub>2</sub> concentrations in the bed is zero along the chromatographic reactor at zero time. Over a period of time, CO and H<sub>2</sub> concentrations in the bed at the inlet of the chromatographic reactor is changed corresponding to the unit step function. And then when more time has elapsed, their concentration profiles along the chromatographic reactor axis does not change significantly. This is the steady state time. If planes of  $z$  equal to one, are interfered to these three dimensional plots, breakthrough curves of outlet concentrations are obtained. These curves are “S” shaped. H<sub>2</sub>O is a byproduct of hydrocarbon products and is consumed with CO in a water-gas-shift reaction to produce H<sub>2</sub> and CO<sub>2</sub>. water-gas-shift reaction in Fe catalysts progresses more

than CO catalysts. But Fe catalysts are cheaper than CO catalyst. In this research Fe-Cu-K catalyst has been used, so water-gas-shift reaction progresses and water is converted to CO<sub>2</sub>. Therefore water concentrations are less than CO<sub>2</sub> concentrations, and this matter is led to reduction in deactivation of Fe catalyst by water. There is not any CO<sub>2</sub> & H<sub>2</sub>O in the fresh feed. Therefore, the concentrations of them are equal to zero at the inlet of the reactor. At the initial time, the bed is full of carrier gas, so the concentrations of CO<sub>2</sub> & H<sub>2</sub>O are zero. Along the  $z$ -axis, they are produced and the concentrations of them are increased. Adsorption of CO, H<sub>2</sub> is less than CO<sub>2</sub> & H<sub>2</sub>O, therefore CO, H<sub>2</sub> have left the reactor before CO<sub>2</sub> & H<sub>2</sub>O. Adsorption together with reaction, decreases water concentration all along the reactor. Therefore, Fe catalyst deactivation is reduced.

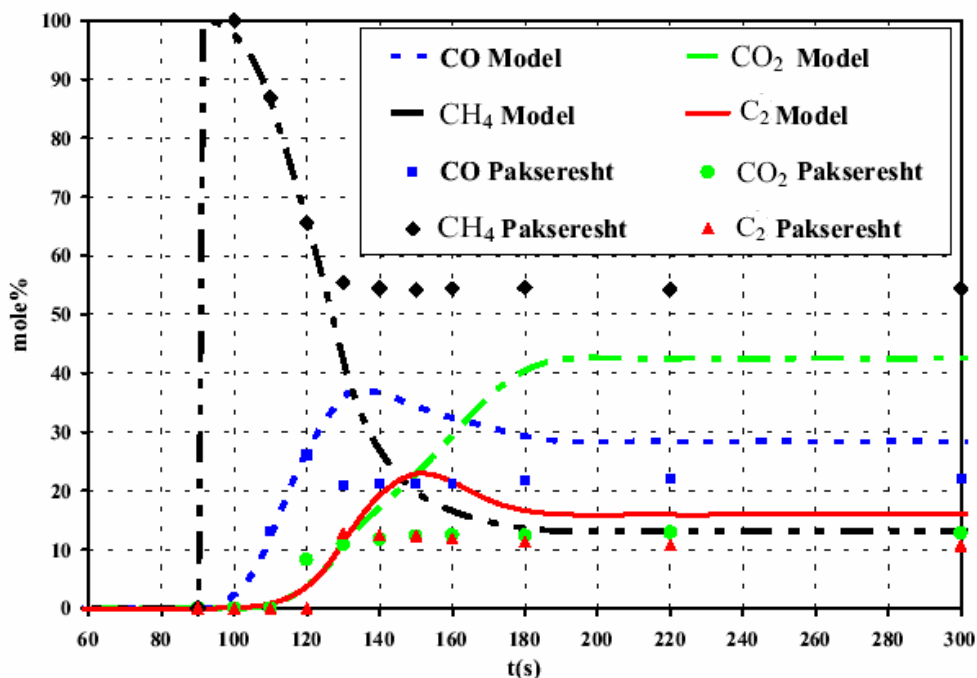


Fig 1. Comparison of present model and experimental results of Pakseresht



**Table 3.** Data that are used for model in Fig. 2a-e

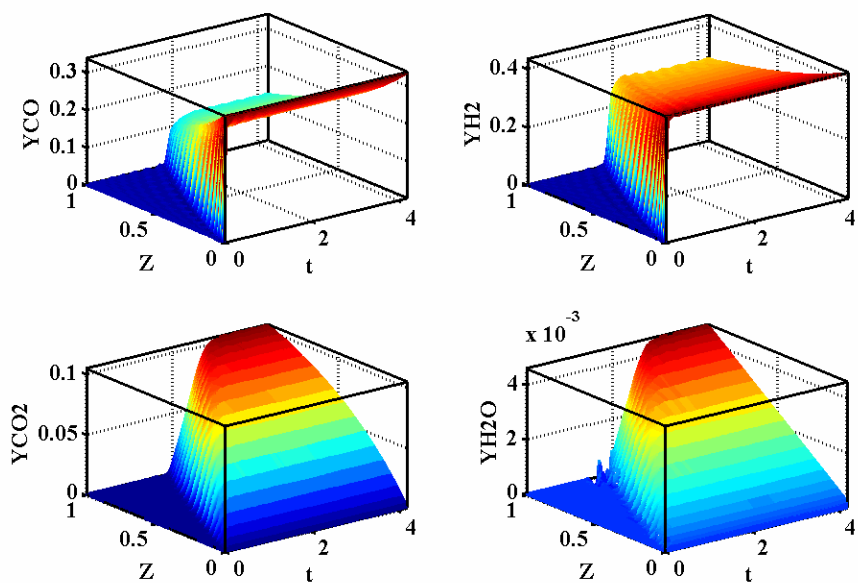
Parameter	Value	Dimension	Description
$\varepsilon_a$	0.2	-	Adsorber volume fraction
$\varepsilon_b$	0.6	-	Fluid volume fraction
$\varepsilon_c$	0.2	-	Catalyst volume fraction
$Y_F \text{ He}$	0.566	-	Carrier gas dimensionless concentration in feed
$Y_F \text{ CO}$	0.189	-	Carbon monoxide dimensionless concentration in feed
$Y_F \text{ H}_2$	0.245	-	Hydrogen dimensionless concentration in feed
$\varepsilon_p$	0.31	-	Adsorbent porosity is used for $k_{Ci}$ correlation
$R_p$	1	mm	Catalyst radius
$T$	280	$^{\circ}\text{C}$	Temperature
$P$	18	Bar	Pressure
$L$	53	Cm	Column length
$D$	2	Cm	Column diameter
$N_{\text{total}}$	22	-	Total no. of collocation points
Alpha	0	-	Jacobian polynomial parameter
Beta	0	-	Jacobian polynomial parameter
$C_{pr}$	1.9762	kmole/m <sup>3</sup>	Reference concentration of the adsorber particle
$C_{br}$	0.2215	kmole/m <sup>3</sup>	Reference concentration of the bed
$v$	0.009	m/s	Superficial velocity

Fig. 2b, contains four plottings of dimensionless concentration of  $\text{CH}_4$ ,  $\text{C}_2\text{H}_2$ ,  $\text{C}_2\text{H}_6$  & He in the chromatographic reactor bed vs. the dimensionless length of the reactor and dimensionless time.

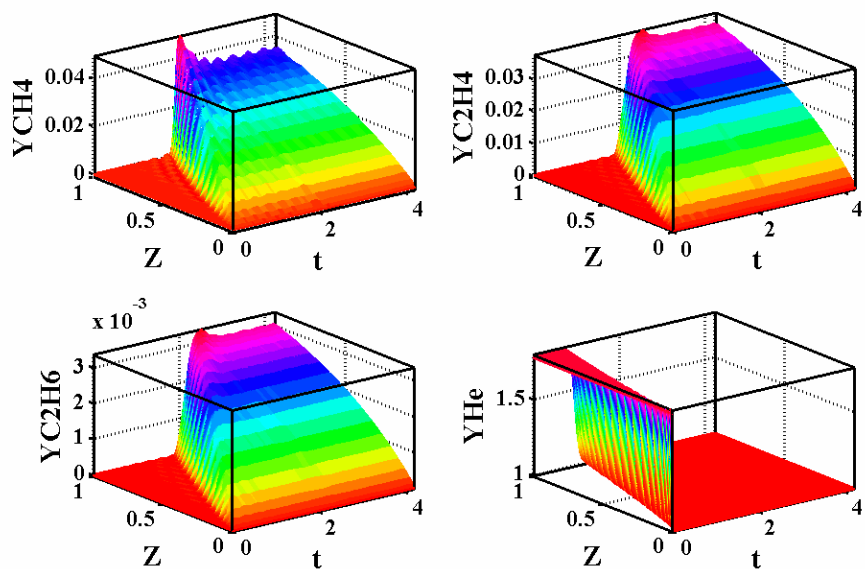
$\text{CH}_4$ ,  $\text{C}_2\text{H}_2$  and  $\text{C}_2\text{H}_6$  are products. They are produced along the chromatographic reactor axis. Ethylene production is more than ethane production and this is good for ethylene production that is one of the important petrochemical feeds. The bed of the chromatographic reactor is full of He at initial time, so the carrier gas concentration is at a maximum level at  $t$  is equal to zero. The concentration of it is reduced to a minimum level and fixed to fresh feed concentration at steady state time. Methane production is not suitable, because it must be converted to syn. gas again. Catalyst kinetics dictates the production of methane, but adsorption helps to separate it in a simulated

moving bed chromatographic reactor because of methane spillage at the outlet of the reactor before steady state time.

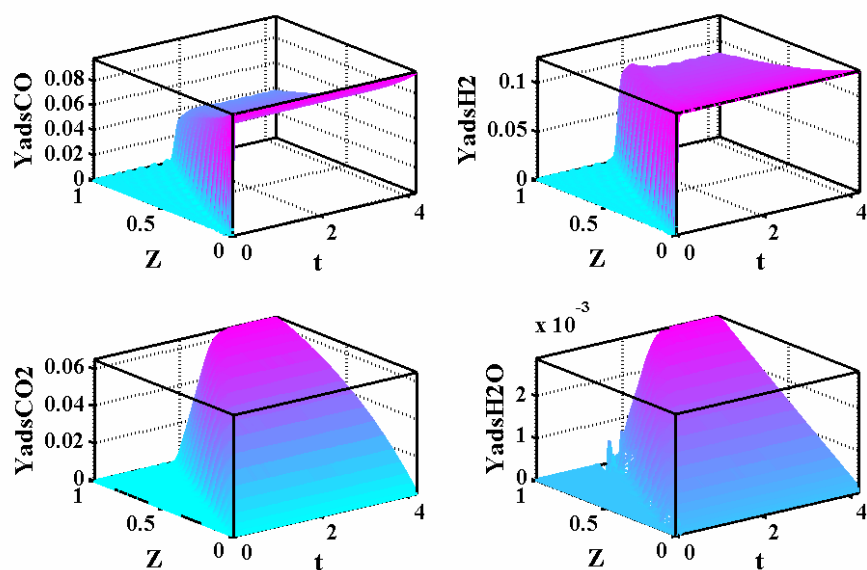
Fig. 2c, indicates three dimensional plots of dimensionless concentration of  $\text{CO}$ ,  $\text{H}_2$ ,  $\text{CO}_2$  &  $\text{H}_2\text{O}$  in adsorber particles vs. the dimensionless length of the reactor and dimensionless time. Concentrations in the adsorber particles follow the concentrations in the chromatographic reactor bed. Increasing concentrations in the chromatographic reactor bed increases the adsorption driving force and increases concentrations in the adsorber particles.  $\text{H}_2$  concentrations in adsorbent vs. time in a constant  $z$  have a maximum. These maximums are dealt with competitive adsorption.  $\text{CO}_2$  &  $\text{H}_2\text{O}$  concentrations in adsorber particles are increased along the  $z$ -axis. Not all the bed becomes full of adsorbed materials because the carrier gas enters with the feed constantly.



**Fig 2a.** Dimensionless concentration of CO, H<sub>2</sub>, CO<sub>2</sub> & H<sub>2</sub>O in the chromatographic reactor bed vs. dimensionless length of the reactor and dimensionless time



**Fig 2b.** Dimensionless concentration of CH<sub>4</sub>, C<sub>2</sub>H<sub>4</sub>, C<sub>2</sub>H<sub>6</sub> & He in the chromatographic reactor bed vs. dimensionless length of the reactor and dimensionless time



**Fig 2c.** Dimensionless concentration of CO, H<sub>2</sub>, CO<sub>2</sub> & H<sub>2</sub>O in the adsorber particles vs. dimensionless length of the reactor and dimensionless time

In Fig. 2d, dimensionless concentration of CH<sub>4</sub>, C<sub>2</sub>H<sub>4</sub>, C<sub>2</sub>H<sub>6</sub> & He in the adsorber particles vs. dimensionless length of the reactor and dimensionless time have been plotted. CH<sub>4</sub>, C<sub>2</sub>H<sub>4</sub> and C<sub>2</sub>H<sub>6</sub> concentrations in the adsorber vs. time in a constant z have maximums. These maximums are dealt with competitive adsorption. The carrier gas, He, does not adsorb.

Mole fractions of some components in the outlet of the chromatographic reactor vs. time have been indicated in two dimensional plots of Fig. 2e. In this chromatographic reactor, upon entering the feed, during the reaction and producing the products, components are separated, because of differences in adsorption selectivity. As Fig. 2e indicates, carbon monoxide and methane exit from the reactor, earlier than carbon dioxide and ethylene. This means that the outlet stream of the FTS chromatographic reactor can be divided into two main streams: one stream contains lighter components (methane and carbon monoxide) and the other stream contains heavier components (carbon dioxide and ethylene). This is the main idea for

developing simulated moving bed chromatographic reactors for reaction and adsorption simultaneously and continuously.

### Conclusions

The fixed bed chromatographic reactor has been modeled for FTS and WGS reactions. The model equations have been solved numerically by the orthogonal collocation method. The trends of the present model predictions and experimental results are in good agreement. Differences between these absolute values might be attributed to the catalyst kinetics which is not the same for both cases. Implementing the axial dispersion model and incorporating the linear driving force approach for the adsorber, resulted in simplicity of the model and a suitable speed for numerical solution. The orthogonal collocation method is suitable for the numerical solution of this model. The model can predict the maximum in breakthrough curves because of competitive adsorption. The model has been run for different conditions and the main conclusions are summarized as follows:

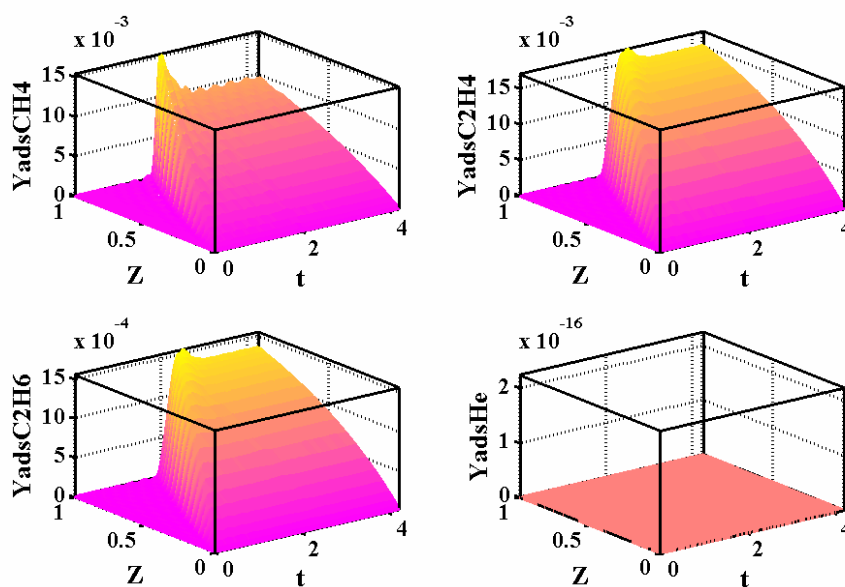


Fig 2d. Dimensionless concentration of  $CH_4$ ,  $C_2H_4$ ,  $C_2H_6$  & He in adsorber particles vs. dimensionless length of chromatographic reactor and dimensionless time

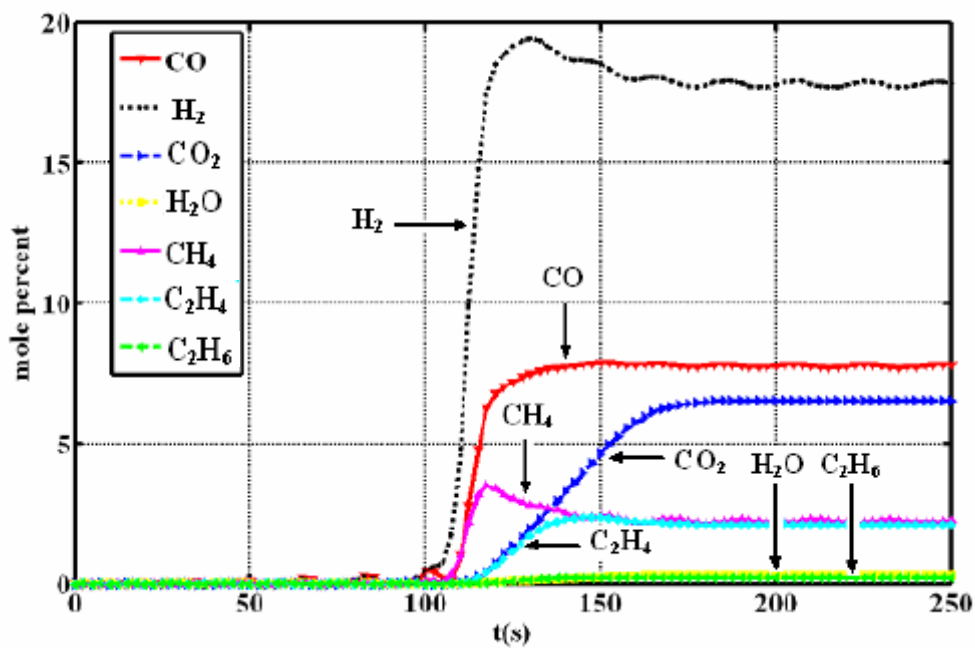


Fig 2e. Mole fractions of some components in outlet of the chromatographic reactor vs. time

- 1-Increasing the temperature can increase the reaction rates and decrease the adsorption rates. Therefore, there is an optimum temperature. In this temperature, suitable adsorption with good reaction progress can be obtained.
- 2-Increase in pressure is suitable for reaction-adsorption and is contrary to desorption.
- 3-Increasing of the volume fraction of absorber advantages the separation.
- 4-Reduction in average velocity and increase in reactor length, increases the reaction and adsorption rates, because the residence time increases.
- 5-By increasing the volume fraction of carrier gas, separation rates are increased and reaction rates are decreased.

The fixed bed chromatographic reactor model developed in this work may be extended to simulate moving bed chromatographic reactors. Therefore, this research presents the basic steps to develop a mathematical model and numerical solution of model equations for a complex reaction and separation phenomenon to be optimized for such important methane conversion processes.

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