CuO/ Water Nanofluid Heat Transfer Through Triangular Ducts

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Abstract
In the present paper laminar flow forced convective heat transfer of CuO/water nanofluid in a triangular duct under constant wall temperature condition is investigated numerically. Sometimes, because of pressure drop limitations the need for noncircular ducts arises in many heat transfer applications. We used nanofluid instead of pure fluid because of its potential to increase heat transfer of system. In this paper, the effect of parameters such as nanoparticles diameter, nanoparticles concentration, type of nanoparticles and heat transfer comparison between nanofluid and pure fluid is studied. Comparison of convective heat transfer of nanofluid in isosceles triangular ducts with various apex angles is also presented. In this study, for the presence of nanoparticles, the dispersion model and for solving differential equations, the finite difference method is used. Numerical results indicate an enhancement of heat transfer of fluid with changing to the suspension of nanometer-sized particles in the triangular duct. Results also defined that equilateral triangular duct has a maximum heat transfer in comparison with other types of isosceles triangular duct.

Keywords: Heat transfer Enhancement, Triangular Duct, CuO/water Nanofluid

1. Introduction
Increased effort is being directed at producing more efficient heat exchangers to effect savings of energy, material and labor. Because of size and volume constraints in applications to aerospace, nuclear, biomedical engineering and electronics, it may be necessary to use non-circular flow-passage geometries, particularly in compact heat exchangers [1]. The optimization of heat exchangers therefore always has to be aimed at an increase in the heat transfer simultaneously with a minimum increase of pressure drop[2]. Consequently, ducts with non-circular cross-section are used in this study due to less pressure drop, although it causes decreasing heat transfer. As the heat transfer rate through the noncircular ducts (triangle, square, rectangle,…etc.) is smaller than that of circular tubes due to less pressure drop, adding nanoparticles to heat transfer fluids may enhance the heat transfer properties of noncircular ducts[3]. An innovative way of improving the heat transfer performance of common fluids is to suspend various types of small solid

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particles, such as metallic, nonmetallic and polymeric particles in conventional fluids to form colloidal. However, suspended particles of the order of μm or even mm may cause some severe problems in the flow channels, increasing pressure drop, causing the particles to quickly settle out of suspension [4]. Nanofluids possess better stability, much higher surface area, less clogging and abrasion [5]. So nanofluid was used instead of pure fluid because of its potential to increase the heat transfer of the system. Nanofluids are created by dispersing nanometer-sized particles (< 100 nm) in a base fluid such as water, ethylene glycol or propylene glycol [6]. In heat transfer applications, the suspension should have a sufficiently high volume fraction of suspended solid, while avoiding significant increases in viscosity relative to the parent liquid. In addition, the suspension should remain stable and avoid sedimentation or degradation during use. The most important requirement is that suspension must be chemically stable, thereby avoiding flocculation, coagulation, or gel formation. Chemical stability can be achieved by use of suitable additives that modify the surface chemistry of the particle–liquid system, or provide a repulsive surface charge, such as that achieved via control of pH [7]. Understanding the physical and thermal properties of nanofluid is essential before using nanofluids in practical applications [7].

Choi [8] was the first person to create fluids containing a suspension of nanometer-sized particles called nanofluids, and indicate their considerable thermal properties by measuring the convective heat transfer coefficient of these fluids. Lee and Choi [9] studied convective heat transfer of laminar flows of an unspecified nanofluid in microchannels, and observed a reduction in thermal resistance by a factor of 2. Nanofluids were also observed to be able to dissipate a heat power three times more than pure water could do. Xuan and Rotzel [10] considered two models (homogeneous and dispersion model) for investigating forced convective heat transfer.

Nanofluids boiling process has been investigated experimentally by several researchers. Bang and Chang [11] studied boiling heat transfer characteristics of nanofluids with alumina nanoparticles suspended in water. They found that the addition of alumina nanoparticles caused a decrease of pool nucleate boiling heat transfer. There are different and opposite parameters affecting the boiling heat transfer performance of nanofluids including the viscosity of the solution, nanoparticle collision with the heater surface and bubbles, and the boundary layer thickness. Soltani et al investigated Pool boiling heat transfer of non-Newtonian nanofluids. The combination of the variations in such parameters causes better performance for non-Newtonian nanofluids in comparison with the non-Newtonian base fluid [12].

There are many passive cases about nanofluids that are still unrecognized. Most of the searches are about heat transfer in circular ducts and there is no report about ducts with a triangular cross-section which causes a lower pressure drop than other forms of ducts.

Kays and London [13] showed that a compact heat-exchanger, with a triangular cross-sectional internal flow passage, has a
high ratio of heat-transfer area to flow-passage volume. Shah and London [14] studied the heat transfer characteristics of laminar flow in a wide variety of channel shapes, including for equilateral triangular with rounded corners, isosceles triangular, right triangular and arbitrary triangular cross-section ducts, for an extensive range of thermal boundary conditions. As can be seen, investigations are about pure fluids, so studying the laminar flow forced convective heat transfer of nanofluid in a triangular duct with constant wall temperature using the dispersion model is the aim of this paper.

2. Mathematical modeling

Laminar flow forced convection of CuO/water nanofluid in a triangular duct is studied numerically. The duct configurations and coordinate system are shown in Fig. 1.

For the hydrodynamically developed and thermally developing flow, there is only one nonzero component of velocity \( u \), and the dimensionless velocity for triangular ducts using dimensionless parameters including \( U = \frac{u}{u_m} \) (\( u_m \) is average velocity) , \( Y = \frac{Y}{2b} \) and \( X = \frac{X}{2a} \) is defined as follows [14]:

\[
U = \frac{15}{b^2} \left[ -b^2 Y^3 + 3a^2 Y X + \left( b^2 Y^2 + a^2 X^2 \right) - \frac{4}{27} b^2 \right]
\]

(1)

The energy equation for constant property flow is defined as:

\[
\frac{k_{\text{eff}}}{\rho c_p} \left( \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) \right) = u \frac{\partial T}{\partial z}
\]

(2)

\( k_{\text{eff}} \) in the above equation is effective thermal conductivity of nanofluid and may take the following form [15]:

\[
k_{\text{eff}} = k_{\text{nf}} + k_d
\]

(3)

\( k_d \) is the dispersion thermal conductivity and the following formula has been proposed to calculate \( k_d \) [15-16 ]:

\[
k_d = C (\rho c_p)_{nf} u_m \cdot v_d \cdot a
\]

(4)

In which \( C \) is an unknown constant and should be determined by matching experimental data.

At the end, the energy equation for laminar flow in an equilateral triangular duct is:

\[
2 a U \frac{\partial T}{\partial Z} = \left[ \frac{k_{\text{nf}}}{(\rho c_p)_{nf} u_m} + \frac{C (\rho c_p)_{nf} v_d a}{(\rho c_p)_{nf}} \right] \frac{\partial^2 T}{\partial X^2} + \left( \frac{a}{5} \right)^2 \left[ \frac{k_{\text{nf}}}{(\rho c_p)_{nf} u_m} + \frac{C (\rho c_p)_{nf} v_d a}{(\rho c_p)_{nf}} \right] \frac{\partial^2 T}{\partial Y^2}
\]

(5)

In Eq. (5), Peclet number can be used to simplify the equation

\[
p_{\text{enf}} = \frac{2 a u_m (\rho c_p)_{nf}}{k_{\text{nf}}}
\]

(6)
Consequently the temperature distribution equation is in the form of:

\[
2\alpha T \frac{\partial^2 T}{\partial x^2} + \left( \frac{2a}{p_{\text{nf}}} + C_{\text{d}} \alpha \right) \frac{\partial^2 T}{\partial y^2} + \left( \frac{a}{b} \right)^2 \frac{2a}{p_{\text{nf}}} + C_{\text{d}} \alpha \frac{\partial^2 T}{\partial y^2} \right] = \rho \nu C_p \frac{\partial T}{\partial \tau} + \frac{\partial}{\partial \tau} \left( \rho \nu \right) = \frac{\partial}{\partial \tau} \left( \rho \nu \right)
\]  

(7)

3. Thermo physical properties of nanofluids

It is expected that the heat transfer coefficient of the nanofluid will depend on the thermal conductivity and the heat capacity of the base fluid and nanomaterials, flow pattern, Reynolds and Prandtl numbers, temperature, the volume fraction of the suspended particles, the dimensions and shape of the particles [17]. So, some of the thermophysical properties used in this paper are defined as:

Density of nanofluid [10, 15, 18]:

\[
\rho_{\text{nf}} = \nu \rho_p + (1-\nu) \rho_{bf}
\]  

(8)

Specific heat capacity of nanofluid [10,15,18]:

\[
C_p_{\text{nf}} = \frac{\nu C_p + (1-\nu) \rho_{bf} C_{Pbf}}{\rho_{nf}}
\]  

(9)

Thermal conductivity is an important parameter in the field of nanofluid heat transfer[19]. Various models for Conductive heat transfer coefficient of nanofluids are proposed. In absence of experimental data, Yu and Choi correlation [20] was used for determination of nanofluid effective thermal conductivity:

\[
k_{nf} = \left[ \frac{k_p + 2k_{bf} + 2(k_p - k_{bf})(1 + \beta)^2 \nu}{k_p + 2k_{bf} - (k_p - k_{bf})(1 + \beta)^2 \nu} \right] k_{bf}
\]  

(10)

In equation (10) \(\beta\) is the ratio of the nanolayer thickness to the original particle radius and \(\beta = 0.1\) was used to calculate the nanofluid effective thermal conductivity.

All the thermophysical properties discussed above were incorporated in the present numerical analysis to compute the laminar heat transfer for three different types of nanofluids which are summarized in Table 1.

<table>
<thead>
<tr>
<th>Nanoparticle</th>
<th>(\rho_p (\text{kg/m}^3))</th>
<th>(C_p (\text{J/kgK}))</th>
<th>(k_p (\text{W/mK}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL(_2)O(_3)</td>
<td>3700</td>
<td>880</td>
<td>46</td>
</tr>
<tr>
<td>CuO</td>
<td>6350</td>
<td>535.6</td>
<td>69</td>
</tr>
<tr>
<td>Cu</td>
<td>8940</td>
<td>385</td>
<td>397.5</td>
</tr>
</tbody>
</table>

4. Validation of the simulation

In this paper, the finite difference method is used for numerical solution. Fig. 2 shows the grid distribution of the triangular duct that was used. The discretization in the physical space \((x, y)\) is performed by dividing the flow domain in equal triangular elements. The grid is constructed by drawing inside the triangular cross section, three groups of parallel lines. The lines of each group are equally distanced and parallel to one of the three sides of the triangle.

The benefits of such a discretization is obvious since the boundaries of the computational domain are identical to the boundaries of the triangular cross section of
the channel, providing good accuracy in the numerical solution[21].

Of course, the involved computational effort is significant since solving for the unknown distribution function, in a general-geometry problem, would require a six-dimensional phase space grid (three variables in the physical space and three variables in the molecular velocity space), which imposes severe demands on computer resources (time and memory) [22]. The grid used in the present analysis is 56 × 56 × 100 (56 in x, y direction, 100 in z direction). In order to ensure grid independence, the solution is tested for 80 × 80 × 180, which gave similar values. Therefore, 56 × 56 × 100 was accepted as the optimal grid size. In order to validate the computational model, the numerical results were compared with the theoretical data available for the conventional fluids in triangular ducts by London [14]. Fig. 3 displays the comparison of Nusselt number computed by London and computed values from the present simulations.

5. Results and discussion
The numerical code developed is used to investigate the effect of parameters such as nanoparticles concentration (ν), nanoparticle diameter (dp), and Reynolds number on the heat transfer of nanofluids. It is also used for Heat transfer comparison between isosceles triangular ducts with various apex angles. Considering the laminar flow regime, the range of Reynolds number is between 100-2300.

Fig. 4 shows the average Nusselt number versus Re for pure water and water/ CuO nanofluid. As shown in Fig. 4, the slope of Nu versus Re is greater for water/ CuO compared to pure water, which means a considerable enhancement of heat transfer by adding nanoparticles to the base fluid. The actual mechanism behind this enhancement remains unclear [23]. For example, at Re=2065, Nusselt number of water is increased from 3.79 to 5.38 by adding nanoparticles of CuO (.01 volume concentration, diameter of 10 nm).

Fig. 5 shows the average Nusselt number versus Reynolds at various concentration of CuO for 10nm-40nm nanoparticles. The effects of nanoparticle size and particle concentration on the thermal conductivity are shown in this figure. This figure indicates that...
Essentially, adding more nanoparticles to the base fluid resulted in the further enhancement of the thermal properties of the base fluid. For example, at \(dp=10 and Re=424\), by increasing nanoparticles concentration from 0.01 to 0.04, the average Nusselt number increases from 2.72 to 3.49 or at higher Reynolds number (\(Re=2123\)), the Nusselt number changes from 5.46 to 6.61. It can be seen that the Nusselt number enhancement by nanoparticles concentration is negligible at high nanoparticles diameters. But Fotukian and Nasr Esfahany [25] showed that in turbulent regime, increasing the nanoparticles concentration did not show much of an effect on heat transfer enhancement in the range of concentrations studied in that work.

**Figure 4.** Comparison between nanofluid and pure fluid heat transfer

**Figure 5.** The influence of CuO nanoparticles volume concentration on the Nusselt number over a range of Reynolds numbers with 10 -40 nm diameter nanoparticles
As an example, by increasing nanoparticle size from 10 to 50nm in 0.02 concentration at Re=1744, the average Nusselt numbers decrease from 5.23 to 4.44. Also, at Reynolds number 2050 in 0.02 suspensions, increasing nanoparticle size from 10 to 50nm leads to a decrease in Nu from 5.68 to 4.82.

This figure indicates that the better enhancement is seen at higher Reynolds numbers. The results illustrate that by increasing nanoparticle concentration from 0.01 to 0.04 at Re = 500, the average Nusselt number increases from 2.88 to 3.6; while at Re= 2000, the Nusselt number changes from 5.3 to 6.23. But it was shown that in turbulent regime, the ratio of convective heat transfer coefficient of nanofluid to that of pure water decreased with Reynolds number[25].

In order to compare the heat transfer enhancement using different kind of solid nano-additive, Fig. 6 indicates the Nusselt number versus Reynolds of 3 nanofluids at various volume fraction for 10nm nanoparticles. As shown in Fig. 6, at dp=10nm and Re=2000, by increasing nanoparticles concentration from 0.01 to 0.04, the average Nusselt number of water/Cu, water/CuO and water/Al₂O₃ increases from 5.79 to 6.51,5.29 to 6.23 and 4.83 to 5.95, respectively. So water/Cu nanofluid with 0.04 volume concentration of 10nm Cu nanoparticles has a maximum heat transfer in comparison with the 2 other types of nanofluid mentioned previously.

It has long been known that the layered molecules are in an intermediate physical state between a bulk liquid and a solid [26], the solid-like nanolayer of liquid molecules would be expected to lead to a higher thermal conductivity than that of the bulk liquid [27]. Because heat transfer between the particles and the fluid take place at the particles and fluid interface [16], equation (10) considers the interface between nanoparticles and liquid (ordered nanolayer at solid/liquid interface) as a parameter for calculating nanofluid thermal conductivity under the static condition. By taking into account the increase in thermal conductivity of nanofluid, other factors such as dispersion and chaotic movement of nanoparticles, Brownian motion and particle migration must be considered in the interpretation of heat transfer performance of nanofluids. Moghadassi et al. [19] studied a model for the prediction of the effective thermal conductivity of nanofluids based on dimensionless groups. They found that the modeled effective thermal conductivity increases as particle size is reduced. This phenomenon is due to the relative effects of nanoparticle motion mechanisms of dilute suspensions such as Brownian motion, thermophoresis and osmophoresis, including
size dependence, on the thermal conductivity. Also, it may be due to the effect of effective surface increasing with particle size decreasing. This is achieved using the dispersion model to analyze heat transfer enhancement of nanofluids.

Fig. 7 shows the average Nusselt number as a function of half apex angle of isosceles triangular duct of this study. As shown in Fig. 7, by increasing the apex angle from 10 to 60, for .01 volume concentration and 10nm CuO particles diameter, Nusselt number and Reynolds number increase from 1.64 to 5.46 and 322 to 2123, respectively. Consequently for laminar flow, nanofluids through equilateral triangular cross-section ducts have a maximum Nusselt number in comparison with other types of isosceles triangles.

**Figure 7.** Heat transfer comparison between isosceles triangular ducts with various apex angles

**6. Conclusions**

In this paper, Laminar flow forced convection of CuO/water nanofluid in a triangular duct is studied numerically. Results indicate that adding nanoparticles to the base fluid increases the heat transfer coefficient of the fluid. Dispersion and random movement of nanoparticles inside the fluid change the structure of the flow field and lead to heat transfer enhancement. This is achieved using the dispersion model to analyze heat transfer enhancement of nanofluids. The results obtained by numerical solution show that decreasing the nanoparticles size increases the Nusselt number at a specific concentration and increasing the nanoparticles concentration increases the Nusselt number at constant particle size. Specifically, the increase in thermal conductivity is relatively independent of particle interaction for low concentrations, while it is strongly dependent on the particle interaction for high concentrations. Results also show that equilateral triangular ducts cause higher heat transfer coefficient than other types of isosceles triangular duct.

**Nomenclature**

- $C_{\rho f}$: Specific heat of fluid [J/kg K]
- $C_{\rho nf}$: Specific heat of nanofluid [J/kg K]
- $C_{\rho p}$: Specific heat of nanoparticles [J/kg K]
- $d_p$: Nanoparticles diameter [m]
- $k_{bf}$: Thermal conductivity of fluid [W/m K]
- $k_{nf}$: Thermal conductivity of nanofluid [W/m K]
- $k_p$: Thermal conductivity of nanoparticle [W/m K]
- $k_d$: Dispersion thermal conductivity [W/mK]
- $k_{eff}$: Effective thermal conductivity [W/mK]
- Re: Reynolds number of nanofluid
- $T$: Nanofluid local temperature [K]
- $T_i$: Inlet temperature of nanofluid [K]
\( T_w \)  Triangle wall temperature [K]
\( u \)  Local axial velocity [m/s]
\( u_m \)  Average axial velocity [m/s]
\( U \)  Dimensionless velocity [m/s]
\( \theta \)  Dimensionless temperature
\( \rho_{nf} \)  Density of nanofluid [kg/m\(^3\)]
\( \rho_f \)  Density of fluid [kg/m\(^3\)]
\( \rho_p \)  Density of nanoparticles [kg/m\(^3\)]
\( \nu \)  Volume fraction of nanoparticles

References


