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# Investigation of Bubble Characteristics by the Photographic Method

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**ABSTRACT**

*The majority of industrial operating units come into contact with two gas and liquid phases. Various methods have been used to determine bubble characteristics over the last few years. In the present study the bubble charactistics have been studied by the shadowgraphy method under the low light condition and the overall mass transfer coefficient have been determined in different pressures and flows. The study has been done in an industrial scale by scrubbers that remove hydrogen fluoride (HF). Using the RLI (reflected LED image) of circularly organized LED light sources created on the bubble surface in low-light situations, a new method for measuring the diameter of the bubble is provided. A system for absorbing gas bubbles by liquid phase has been designed in this project. An image analysis method and empirical relations were used to investigate the mass transfer and hydrodynamic behavior in the wake of single air bubbles rising. The overall properties of the bubble, including the size, shape, path, rising velocity, and mass transfer coefficient of a single bubble were studied and measured using these methods. The investigation was carried out in different liquids using a 0.15 × 0.15 × 0.35* $m^3$ *bubble column and nozzles with the diameters of 0.5, 1, 1.5, 2, 2.5 mm. The results show that increasing the diameter of the nozzle increases the diameter of the bubble, resulting in a decrease in velocity. Furthermore, increasing the viscosity of the liquid phase indeed causes the diameter of the bubble to increase while decreasing the velocity. So, based on these findings, we can conclude that the diameter of the bubble is affected by the physical properties of the fluid and has a direct relationship with the diameter of the nozzle.*

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## 1. Introduction

One of the most important and widely used

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phenomena in chemical engineering, including in water treatment and biological processes, is a mass transfer from gas to liquid. A variety of different approaches have been used for determining the properties of bubbles and mass transfer rates. Bubbles are characterized based on their size, trajectory, shape, rise velocity, and mass transfer rate. In this literature, we characterized the bubble dynamics using a photographic method. The properties of the gas and liquid and the bubble production process are among the factors that affect the bubble size. In literatures, including [2, 3], many equations have been proposed to estimate the bubble size. The Bubble velocity not only is one of the important considerations of designing a mass transfer system, but also plays a very important role in calculating the mass transfer coefficient and is very necessary to calculate the bubble residence time. Meanwhile, two cameras were used for capturing the images of the bubble at two different heights through the photographic method [4]. There are several studies that have been used fluorescent dyes with the PLIF (Planar Laser Induced Fluorescence) technique for the visualization of the bubble wake [5-9] and M. Taghavi and J. S. Moghaddas [10] used PLIF/PIV techniques for investigating reactive mixing in stirred tank reactors. The present study is an attempt to extend the previous works by providing different solutions. The first step in this study was to concentrate on the different characteristics of a bubble including size, velocity, and trajectory, after that we determined the mass transfer by using empirical relations R. Higbie and N. Frossling [11, 12] and penetration theory. The total mass transfer depends on the difference between two points of equilibrium. But another important point is how fast this mass is transmitted, and this is what the mass transfer coefficient suggested. There were numerous proposed empirical relations that included factors such as the bubble size, presence or absence of surfactants in the liquid phase, type of the liquid phase flow, etc for expressing the mass transfer rate. In the final step, we calculated the Reynolds (Re), Morton (Mo) Weber (We), Froude (Fr), and Eotvos (Eo) dimensionless numbers. The Reynolds number is the ratio of inertial forces to viscous forces. The Morton number describes the properties of the fluids, for instance, in the water fluid, the value is $10^{-11}$. The Weber number is the ratio of inertial forces to surface tension forces. The Eotvos number is a ratio of the buoyancy Forces to the surface tension force and is often used to characterize the shape of the bubble.

## 2. Experiments

Figures 1 and 2 show the experimental setup created with a high-speed camera. The setup has been placed in a space (2 $m^2$) where we have covered all the walls with black material to block out light distractions. On top of the setup, an LED lamp has been used to provide light. The camera is located across from the setup. The air was distributed into the setup from the bottom using orifices of various sizes. The pressure of air was controlled before approaching the setup area. The column is a square cross-section of $0.15 \times 0.15$ $m^2$ and 0.35 m height that was constructed from plexiglass with a thickness of 6 mm. The single bubbles were produced by orifices with diameters 0.5, 1, 1.5, 2, and 2.5 mm. All measurements were made at atmospheric pressure and room temperature. The liquid phase was filled up to 25 cm in height. The experiments were carried out with 5 different solutions (Table 1). The size, trajectory detection, rising velocity, mass transfer rate, and dimensionless numbers of the bubbles were computed by the common image processing with a digital camera. The digital camera model Hdr-Xr260 with a memory of 160 GB that produced Full-Hd videos was made by Sonny company. We used different software (Aoao video, Video studio…) for converting videos to consecutive photos. The analysis of the bubble size was done through raw images and that of the velocity was done through the videos.

First, experiments were done for tap-water in nozzles with five different diameters. The air was injected to the setup by the compressor, then the video of trajectory of the bubble was recorded by camera and after converting the video to photos, different parameters were detected.

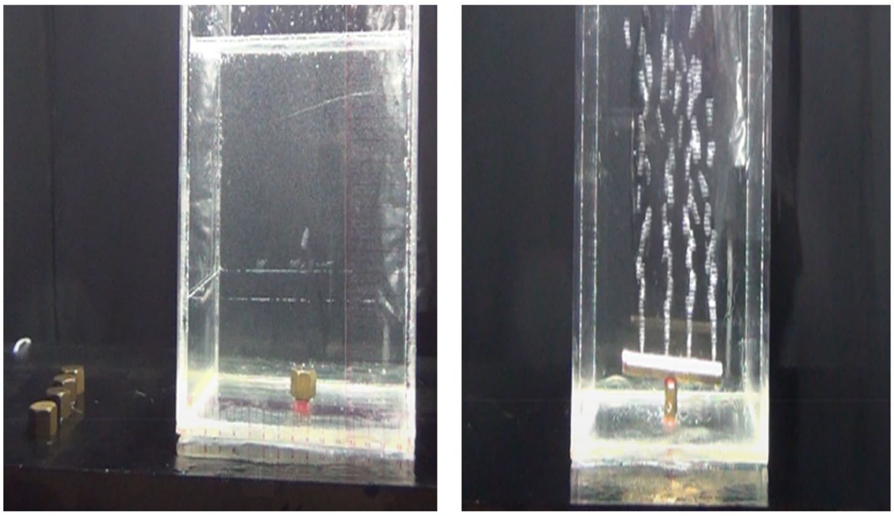


**Figure 1.** Experimental set-up.

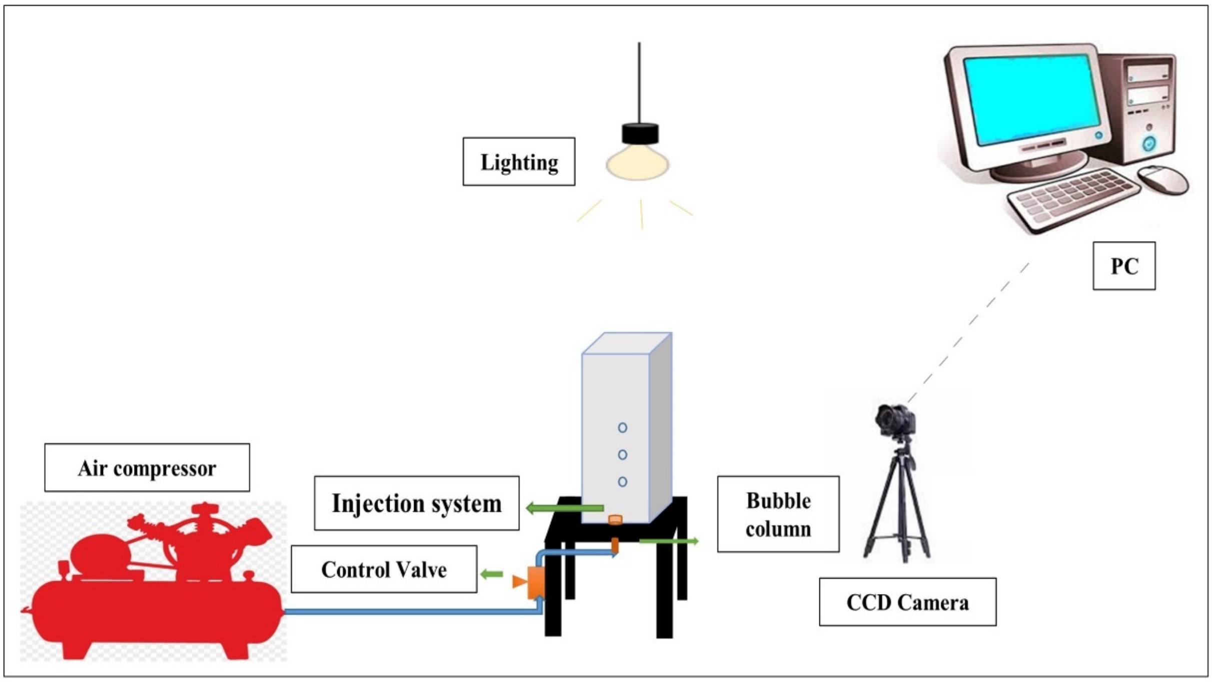



**Figure 2.** Experimental set-up for the photographic measurement.

**Table 1**
Experimental fluids.

| No. | Fluids | Chemical formula | Molecular weight (gr/mol) | Density (kg/m$^3$) | Viscosity (pa.s) |
|---|---|---|---|---|---|
| 1 | Tap-water | $H_2o$ | 18.0153 | 998 | 0.001 |
| 2 | Distilled water | $H_2o$ | 18.0153 | 992 | 0.001 |
| 3 | Ethanol | $C_2h_5oh$ | 46.07 | 789 | 0.0012 |
| 4 | Methanol | $Ch_3oh$ | 32.04 | 791 | 0.00059 |
| 5 | Glycerol | $C_3h_8oh$ | 92.09 | 1267 | 1.412 |

## 3. Results and discussion

### 3.1. Diameter, trajectory & velocity of bubbles

#### 3.1.1. Diameter of bubbles

Three basic categories of single-rising bubbles have been established by W. J. Nock [13]. For the bubble with the diameter of $d_e < 1$ mm, the dominant force is the surface tension force and the bubble shape is spherical and the viscous and interfacial forces are proportionally large. But for an intermediate-sized bubble with the diameter of 1 mm < $d_e$ < 15 mm, the bubble shape is ellipsoidal and without symmetry in back and front. These medium-sized bubbles are affected by inertia forces and surface tensions. Larger bubbles with a spherical cap have oscillating shapes and concave bases. The results of the present study indicate that bubbles are in medium part and have ellipsoidal shape. The bubble generation process and properties of the gas and the liquid are the most important parameters which affect the bubble size. The results of this study show that for all explained fluids, by increasing the diameter of the nozzle, the diameter of the experimental single bubble was increased. And in the studies of E. Kosari et al. [14] it has been shown that the diameter of the bubble at the departure increases as the diameter of the needle, viscosity of the liquid, and gas flow rate increase. The equations of K. Akita and F. Yoshida [2] and T. K. Sherwood et al. [3] for bubble size have been used in this literature. By comparing the results of these two empirical relationships, we can conclude that the result of the work of Sherwood et al. is closer to the actual experimental diameter of the bubbles and also the fact that increasing the diameter of the nozzle increases the diameter of the initial bubble in each cases. The velocity of the bubble has decreased as the diameter of the nozzle has increased. As a result, we can conclude that increasing the diameter of the bubble increases the pressure differential between the front and back of the bubble, resulting in the increased drag force and decreased velocity of the bubble. The results related to the glycerol solution are depicted in Figure 3. The analysis indicates that for the nozzle with the diameter of 0.5 up to 2.5 mm the diameter of a single bubble is 2.6 up to 3.8 mm. These results are comparable with the results of P. Kovats [9], because he has given the diameter of the bubble of 2.2 up to 3 mm for the diameter of the nozzle of 0.25 mm.

Akita & Yoshida, (1974):

$$d_e = 1.88 d_{or} \left( \frac{u_{or}}{\sqrt{g d_{or}}} \right)^{\frac{1}{3}} \tag{1}$$

Sherwood et al., (1975):

$$d_e = \left( \frac{6 d_{or} \sigma}{\Delta \rho g} \right)^{\frac{1}{3}} \tag{2}$$

### 3.1.2. Bubble's trajectory

Bubbles in highly viscose solutions have straight directions and show less fluctuation but on the other hand, they have a zigzag trajectory and more fluctuations in slightly viscose solutions. Figure 4(a) indicates the bubble's trajectory in the methanol solution and Figure 4(b) shows a bubble's trajectory in the glycerol solution.

### 3.1.3. Velocity

Different parameters affected the rising velocity of the bubble. The most influencing parameters are the size and shape of the bubble, properties of the gas and the liquid, bubble generating process, presence or absence of surfactant (clearness or contamination), internal circulation, bubble releasing method and temperature. The internal circulation reduces the viscous drag, resulting in a terminal velocity of about one and a half times as compared to the rigid particle of R. Clift et al. [15]. And in the studies of S. Karimi et al. [16] the terminal velocity linearly depends on Reynolds number and for estimating the terminal velocity and drag coefficient, four empirical correlations were developed. By comparing different relations of the increasing velocity relying on the diagram that has been presented by W. J. Nock [13], we could claim that in some cases along with the diameter of the rising bubble, velocity has been decreased and in some cases, it has been increased, so it depends on the different parameters about which we talked in detail. In our research, the rising of the diameter of the nozzle leads to a decrease in velocity and for the orifice with the size of 0.5 up to 2.5 mm, the velocity is 0.1 up to 0.6 m/s. By adding 20 % glycerol to water, the velocity was reduced by 26 % which signifies the effect of viscosity on the velocity of the bubble. In this part, we have compared the diameter and velocity of bubbles in five different solutions. Based on the results, we could claim that the diameter of the bubble in the glycerol solution is at its maximum level and on the other hand, it is at its minimum level in the methanol solution (Figure 5(a)). As it can be seen from Figure 5(b), the result for velocity is quite opposite of that for the diameter of the bubble which means that the minimum amount of the velocity of the bubble is in the glycerol solution, and its maximum amount is in the methanol solution. In the case of empirical relations, W. J. Nock [13] predicted lower rising velocity for the contaminated liquids and higher rising velocity for the clean liquids, at least for the diameters of $d_e < 3.5$ mm.

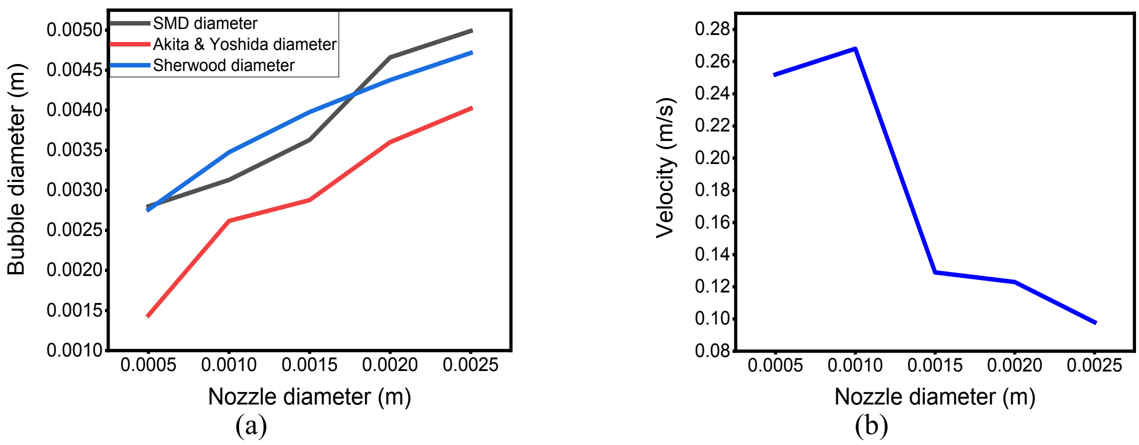



**Figure 3.** Results for the glycerol solution.

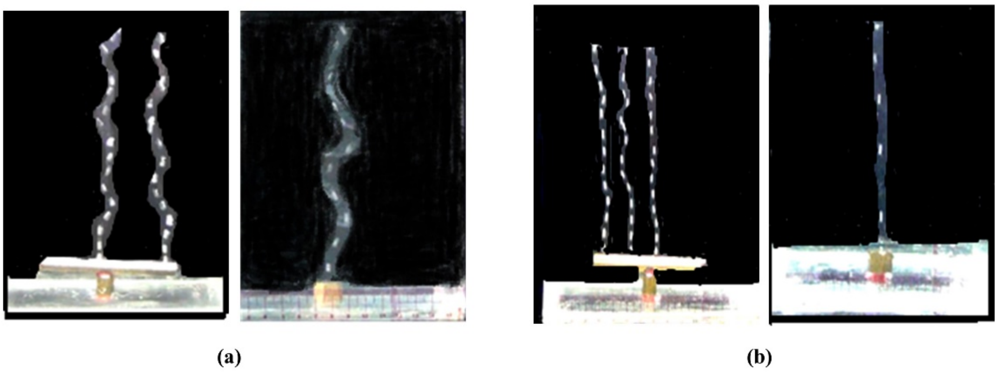



**Figure 4.** (a) Methanol solution, (b) Glycerol solution.

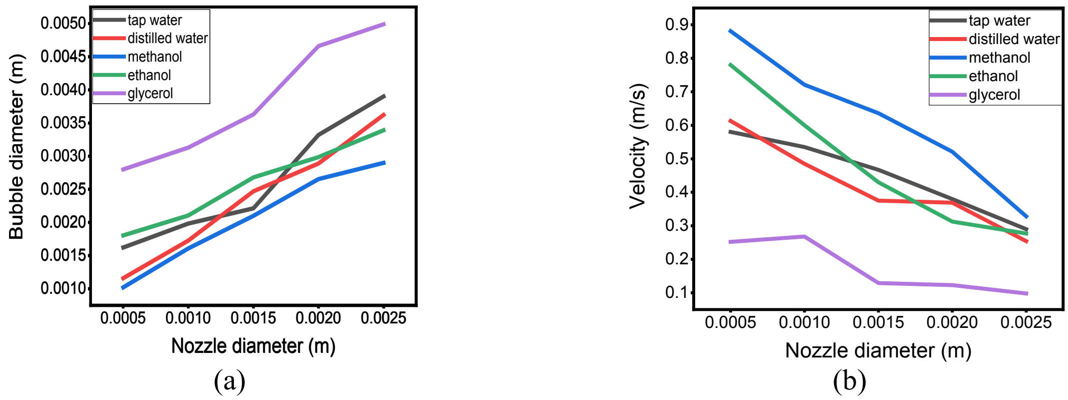



**Figure 5.** (a) Diameter of the bubble according to the diameter of the nozzle, (b) Velocity of the bubble according to the diameter of the nozzle.

### 3.2. Mass transfer

As mentioned below, different empirical relations have been used for determining mass transfer considering different parameters like the bubble size, bubble oscillations, surfactant in the liquid phase, turbulence of the liquid phase, etc.

W. J. Nock [13] declares that, the reduction of the diameter of the bubble ($d_e$ < 2 mm) results in a reduction of the $K_L$ of $CO_2$ bubbles in de-ionized water and synthetic seawater. According to some scientists, oscillations have increased the mass transfer rate [17-21] and indicated a reduction of the mass transfer rate by time. According to W. J. Nock [13], if the experiments were performed at different rising heights, the average $K_L$ would change dynamically due to the intensity of mass transfer. The present study has done investigations by using R. Higbie and N. Frossling, and Pentration theories. As it is obvious, the high mass transfer rate in pure bubbles is expressed by R. Higbie [11] meanwhile the low mass transfer rate in the contaminated liquid phase which causes the bubbles to behave as hard spheres is expressed by N. Frossling [12]. The movable joint surface has a high mass transfer rate and a fixed joint surface has a low mass transfer rate.

Higbie (1935):

$$K_{LH} = \frac{D}{d_b}\left(1.13Re^{0.5}Sc^{0.33}\right) \quad (3)$$

Frossling (1938):

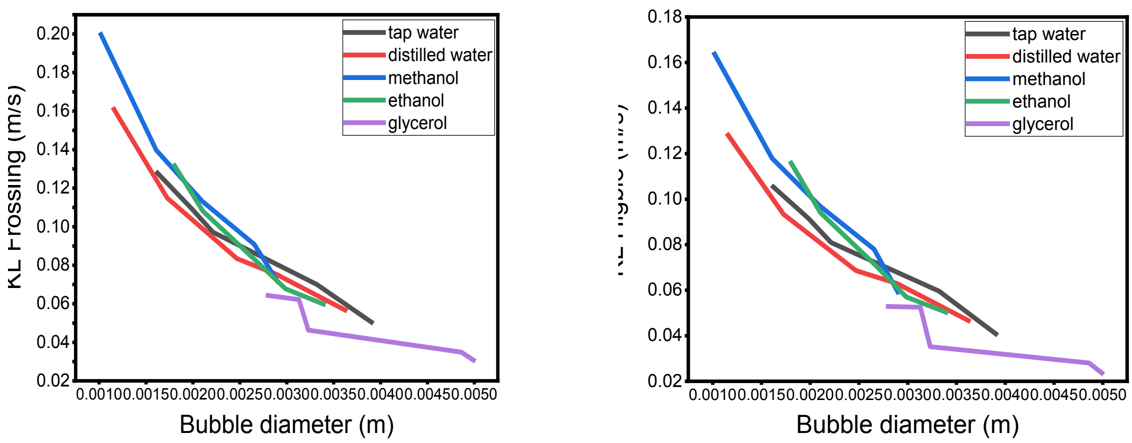
$$K_{LF} = \frac{D}{d_b}\left(2 + 0.6Re^{0.5}Sc^{0.33}\right) \quad (4)$$

The result of our research is depicted in Figure 6, where, as we can see, by increasing the diameter of the bubble, R. Higbie and N. Frossling mass transfer coefficient for all presented fluids have decreased and among the fluids maximum amount belonged to the methanol solution, and the minimum amount to the glycerol solution.

Another method that has been suggested for calculating the mass transfer coefficient is the penetration theory. The study of the motion of the spherical bubble within the liquid phase has been carried out using the penetration theory. In this study, the calculation of this coefficient is carried out with the help of the imaging technique. The analysis is (q: is the time that the bubble needs to pass its diameter) gaining from the height that the bubble has passed in the experimental setup, the time for this rising, and the diameter of the bubble. The mass transfer coefficient has been calculated by assigning q to the penetration relation. The table below shows the calculations of this theory for all five solutions.

Penetration theory:

$$K_L = \sqrt{\frac{D_{AB}}{Pq}} \tag{5}$$




(a) (b)

**Figure 6.** (a) Diameter of the bubble according to $K_L$ Frossling, (b) Diameter of the bubble according to $K_L$ Higbie.

**Table 2**

Result of the penetration theory.

| Nozzle diameter (m) | 20 % Water-glycrel | Tap water | Distilled water | 20 % Water-ethanol | 20 % Water-methanol |
|---|---|---|---|---|---|
| 0.005 | 2.93E-4 | 1.5 E-4 | 1.21 E-4 | 1.34E-4 | 9.46E-05 |
| 0.001 | 3.01E-4 | 1.7 E-4 | 1.66 E-4 | 1.65E-4 | 1.31 E-4 |
| 0.0015 | 4.67E-4 | 1.9 E-4 | 2.26 E-4 | 2.2E-4 | 1.59 E-4 |
| 0.002 | 5.41E-4 | 2.6 E-4 | 2.46 E-4 | 2.72E-4 | 1.98 E-4 |
| 0.0025 | 6.28E-4 | 3.2 E-4 | 3.32 E-4 | 3.08E-4 | 2.61 E-4 |
| average | 4.46E-4 | 2.2 E-4 | 2.18 E-4 | 2.2E-4 | 1.69 E-04 |

The result of our study (Table 2) is comparable with the result of W. J. Nock's [13] that ranging $K_L$ from the maximum value of $5 \times 10^{-4}$ $ms^{-1}$ up to the minimum value of $1 \times 10^{-4}$ $ms^{-1}$. He has plotted the $K_L$ value (from different relations) as a function of the diameter of the bubble from 0 up to 5 mm. According to A. C. Lochiel and P. H. Calderbank [22], for the moving and spherical surfaces, the highest values for $K_L$ will be obtained using the penetration theory, while for stationary surfaces, the values for $K_L$ will be obtained based on the N.Frossling theory.

### 3.3. Dimensionless numbers

Dimensionless numbers in fluid mechanics are the collection of dimensionless quantities that play significant roles in analyzing the behavior of fluids and are expressed as the ratios of the relative magnitude of fluid and physical system characteristics. These numbers can be examined to see whether particular effects or forces should be considered or can be safely ignored in the model. The ratio of the fluid inertial force to the fluid viscous force is expressed by the Reynolds number. Both open and closed surface flows can be described using the Reynolds number. The Morton number, that is independent of the bubble size and velocity, has the advantage of being a property of the gas and the fluid. For air bubbles in water, the Morton number is $Mo = 10^{-11}$. The Weber number expresses the ratio of the particle inertial force to the surface tension force. The ratio of the gravitational force to the inertial force is known as the Froude number. When there are free surface flows and gravitational force is more powerful than other forces, Froude number is important. The proportion of the buoyant force to the surface tension force is represented by the Eotvos number. Although droplets flowing through a gas can also be characterized by this number, it is typically employed in relation to bubbly flows. This quantity is in the order of $Eo = 10^{-1}$–$10^{1}$ when taking into account the bubbly flows of 1-10 mm of the air bubbles rising through water. The shape of the bubble is nearly spherical throughout its ascent for smaller bubbles ($Eo < 10^{0}$), and the drag and lift forces are mostly dependent on the Reynolds number. The drag forces and lift forces are also influenced by the Eotvos number for larger bubbles ($Eo > 10^{0}$), where the surface tension forces are insufficient to maintain a spherical bubble shape. Figures 7, 8, 9, 10, and 11 illustrate the outcome for the Reynolds, Weber, Froude, Morton, and Eotvos numbers. The investigation showed that adding methanol to water enhanced the Reynolds, Weber, Froude, and Morton numbers while decreasing the Eotvos number. Additionally, by adding glycerol to water, Reynolds, Weber, and Froude numbers decreased but Morton and Eotvos numbers have increased. Dimensionless numbers in fluid mechanics are sets of dimensionless quantities that play important roles in analyzing the behavior of the fluids which describe as ratios of the relative magnitude of fluid and physical system characteristics such as density, viscosity, the speed of sound, the flow speed, and etc. Also the results of this study and that of P. Kovats et al. [9] are shown in Table 3 (a, b). P. Kovats has done this calculation just for water but in this essay it has been done for five different solutions. By comparing the results of both studies, we could catch reasonable consistency between the two experiments.

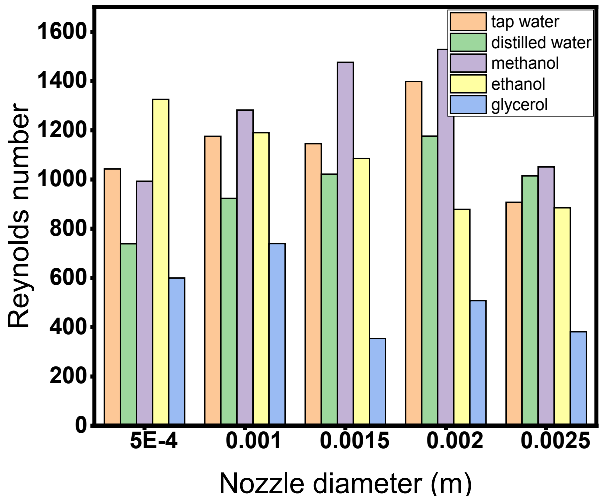



**Figure 7.** Reynolds number for five solutions in nozzles with five diameters.

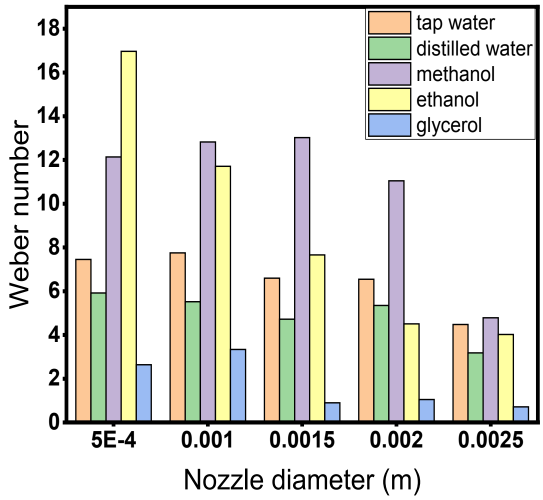



**Figure 8.** Weber number for five solutions in nozzles with five diameters.

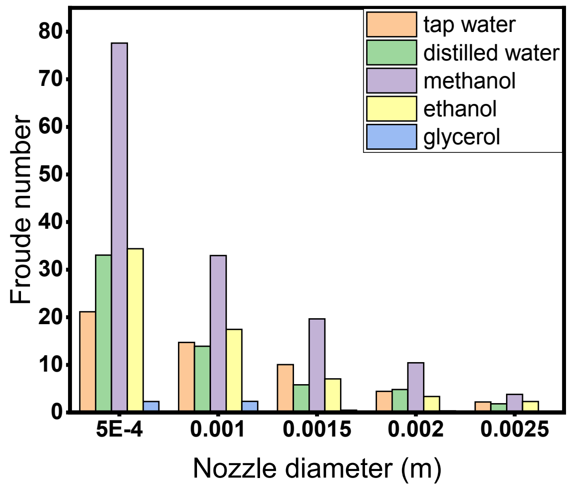



**Figure 9.** Froude number for five solutions in nozzles with five diameters.

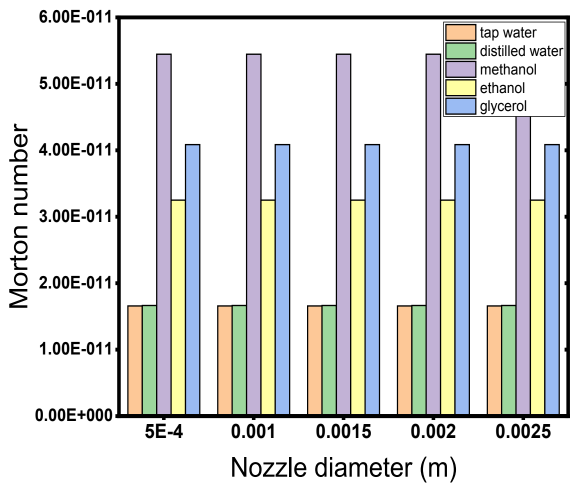



**Figure 10.** Morton number for five solutions in nozzles with five diameters.

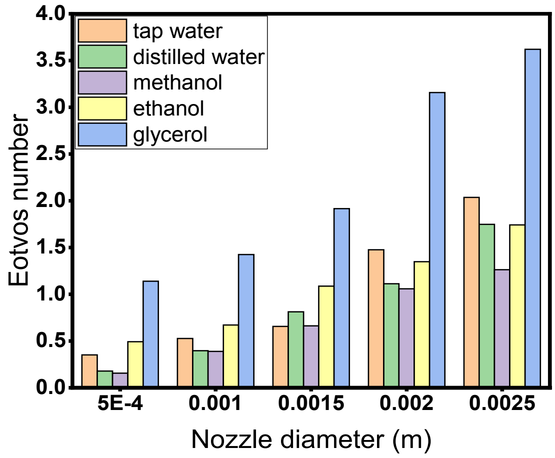
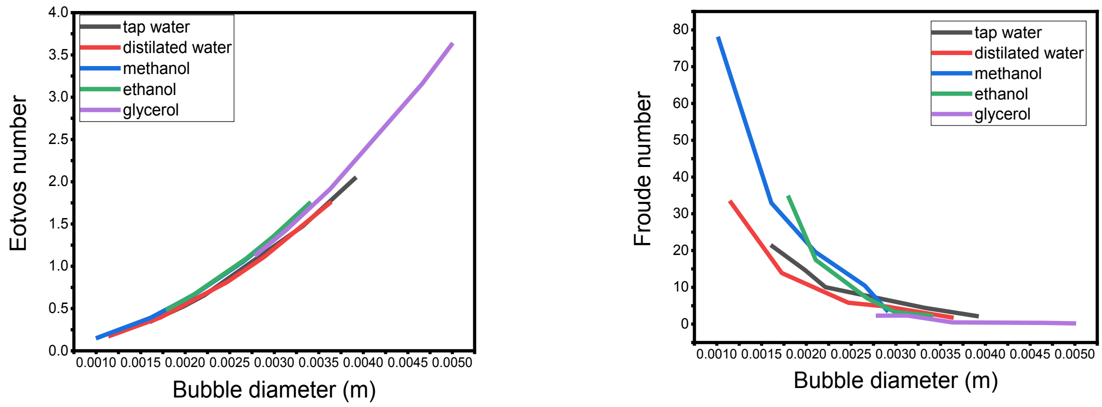



**Figure 11.** Eotvos number for five solutions in nozzles with five diameters.

**Table 3**
Result of dimensionless numbers.

| Bubble diameter | | Reynolds | Morton number around the bubble | Weber number around the bubble | Froude number around the bubble | Eotvos number around the bubble |
|---|---|---|---|---|---|---|
| a | 1.44-6.3 mm | 803 | 2.55E-11 | 3.4 | 3.78 | 0.91 |
| b | 2.8-5 mm | 3.82E+2 up to 1.5E+3 | 1.65E-11 up to 5.45E-11 | 1.05 up to 7.11E+1 | 3.3E-1 up to 3.44E+1 | 6.62E-1 up to 3.62 |

After performing different investigations, as shown in Figure 12, we could claim that by increasing the diameter of the bubble Eotvos number was increased while Froude number was decreased. Bubbles in less viscose solutions, have less Eotvos numbers but high Weber numbers. According to Temos analysis about Reynolds number, bubbles have fluctuations and eddy Penetration mass transfer.




(a) (b)

**Figure 12.** (a) Eotvos number according to the diameter of the nozzle and (b) Froude number according to the diameter of the nozzle.

## 4. Conclusions

The current study used the photographic technique to characterize the fluid dynamics and mass transfer rate. The bubble behavior was examined through various functions such as nozzles with different diameters and different fluids. According to the results of the study, we conclude that, by increasing the diameter of the nozzle, the diameter of the bubble in all presented fluids has been increased and rising velocity has been decreased. Among the five solutions, the glycerol solution has different behavior from others, which has a bubble with bigger diameter and less velocity, Higbie and Frossling mass transfer coefficient. Increasing viscosity has two effects on the hydrodynamics of the bubble. First, it calms the bubble's expansion, and second, it calms the displacement of molecules on the bubble. As we determined in this study, by adding 20 % glycerol to water, the velocity of the bubble decreased 26 % and we discovered a straight pathway in highly viscose solutions and a zigzag path in slightly viscose solutions. Hence, we can conclude that the viscosity of the glycerol solution, has caused this differentiation. So, we can add the conclusion that the viscosity, velocity, diameter of the bubble, density, and surface tension of the solutions have the most influence on the outcomes. The penetration theory is one of the methods used in this study to calculate the mass transfer rate as we can see, the data collected is comparable to data from previous studies and the collected data fits those studies extremely well. The method presented a new insight by combininng the photographic method with theoretical models and will give data for future numerical simulations.

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