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# Predicting the Kinematic Viscosity and Cetane Number of Diesel-Biodiesel Blend using Neural Network and Empirical Models

***M. Yari, Gh. R. Moradi*[*], *M. Abdolmaleki, Sh. Bashiri***

*Catalyst Research Center, Faculty of Chemical and Petroleum Engineering, Razi University, Kermanshah, Iran*

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**ABSTRACT**

*Biodiesel, as a renewable and environmentally friendly fuel, which has gained great popularity in recent years, is a feasible alternative to fossil diesel. However, due to some undesirable properties such as higher viscosity, biodiesel must be blended with diesel in order to be utilizable in a diesel engine. Therefore, a reasonable approach is required for predicting the diesel-biodiesel blend properties. This study tries to estimate two substantial properties of the blend, i.e. kinemattic viscosity (KV) and cetane number (CN), through neural network (NN) and empirical models which use the properties of pure biodiesel (kinematic viscosity, boiling point, evaporation point, flash point, pour point, heat of combustion, cloud point, and specific gravity) as independent variables. In this regard, a three-layer feed-forward network with varying input parameters, training algorithms, transfer functions, and hidden neurons has been examined to predict the KV and CN of the diesel-biodiesel blend. Besides, the prediction capability of thirty empirical equations is investigated to determine the top equations describing the properties of the blend. The result reveals that an ANN with three input parameters of the concentration (%),CN and cloud point of the biodiesel has the best prediction of CN with an R-value of 0.9961. Moreover, NN estimates the KV of the blend with the highest correlation coefficient of 0.9985. The results corresponding to empirical equations also indicate that fractional-exponential equations are the best describer of the CN and KV of the blend with R-values of 0.9947 and 0.9980 respectively.*

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## 1. Introduction

For several decades, fossil fuels have been fundamental drivers of technological, social, and economic growth. However, the depletion

[*]Corresponding author: gmoradi@razi.ac.ir (Gh. R. Moradi)

of fossil fuel reserves, their unfavorable environmental effects, and the increase in fossil fuel prices are the main alarms, which have persuaded governments and scholars to find alternative energy sources [1-4]. Biodiesel, as a feasible and renewable source of energy, is the most potent alternative fuel to fossil diesel that has grabbed the attention of researchers, in recent years [1, 4-6]. Biodiesels are the mono-alkyl esters of long-chain fatty acids and refer to any diesel fuel substitutes that are derived from a variety of natural feedstock such as animal fat, vegetable oil, microalgae, and waste oil [3, 5, 7]. Biodegradability, renewability, availability, higher combustion efficiency, lower toxicity, and having no content of sulfur are worth being mentioned as some of the benefits of biodiesel over the conventional diesel and which make biodiesel a good candidate for the transportation sector. Utilizing biodiesel not only reduces greenhouse gas emissions but also improves the characteristics of diesel engines like brake power, brake specific fuel consumption, and brake torque [2, 5, 6, 8]. Also, due to its complete miscibility, biodiesel can be used as pure or blended with petroleum diesel in a diesel engine without any engine modifications being needed [9, 10]. Although there is no limitation in blending biodiesel with diesel, blending is not sufficient for a higher proportion of biodiesel. The main reason behind this is that the viscosity, density, and price of biodiesel are higher than those of diesel which lead to poor atomization, incomplete combustion, and higher final cost [11-13]. Therefore, it is crucial to determine the properties of the diesel-biodiesel blends experimentally or through mathematical models in order to choose a proper composition that is reasonable to be utilized in a diesel engine. Two main important diesel-biodiesel properties that considerably affect the engine performance are cetane number and viscosity. The cetane number indicates the potential of fuel to auto-ignite in a combustion chamber. A fuel with a higher cetane number shortens the ignition delay and at the same time increases the performance of the diesel engine in terms of engine stability, noise level, and exhaust emissions. It should be noted that biodiesel normally has a higher cetane number (between 45 and 67) than the standard diesel fuel (between 40 and 49) [14, 15]. On the other hand, fuels with lower viscosity are favorable for diesel engines. Lower viscosity facilitates the atomization of the fuel spray and reduces the ignition delay period, while a higher viscosity increases the required energy for pumping fuel, results in poor fuel atomization, and redoes the efficiency of the engine [10, 16-18].

Despite its accuracy, achieving a comprehensive data set of the properties of the diesel-biodiesel blend through an experimental approach is costly in terms of money, time, and effort. So, as a cheaper, fast, and feasible method, a mathematical model is the best alternative to evaluate diesel-biodiesel properties [9, 13, 15]. Empirical equations and artificial neural network models have successfully been used for estimating the physical and chemical properties of diesel and biodiesel in various applications [19-23]. An Artificial Neural Network (ANN) is a computational model of nervous systems of which the capabilities of producing solutions for various linear and nonlinear problems are well documented. A neural network consists of the three main components including an input layer, a processing (hidden) layer, and an output layer, however, NN has other influencing components such as neurons, transfer function, and activation function that

significantly affect its performance. It is worth mentioning that there is no definite method to determine a network's components, and most of the time they are selected by the trial-and-error method [12, 24-26]. Alternatively, using empirical equations is another popular method for predicting the properties of the diesel-biodiesel blend. They are mathematical equations of which free parameters are specified by a given set of experimental data. An empirical mathematical model makes a relationship between one dependent variable and a set of other independent variables; thus allowing new data to be predicted.

Several studies can be found in the published works that have developed mathematical approaches to predict the characteristics of the blend. For example, Ibham, et al. [15] examined and predicted the cetane index of biofuel-diesel blends using both empirical mathematical and artificial neural network models. This study discovered that a feedforward backpropagation network with 4 input, 10 hidden, and 1 output neurons is the best predictor. Furthermore, they showed that, among empirical mathematical models, a 3$^{rd}$ order polynomial model is the best model to predict the cetane index. In another study, Fernanda et.al [27] reliably predicted the cetane index, flash point, and sulfur content of the diesel-biodiesel blend using an artificial neural network. Mert and Atilla [28] performed an experimental study to determine the density and viscosity of various diesel-vegetable oil binary blends at diverse temperatures (278.15-343.15 K) and subsequently predicted the experimental data using empirical models. They developed new rational and exponential models based on temperature to estimate the experimentally measured data. According to their results, for the viscosity data, the best correlation was achieved by the rational model and taking into account the lowest maximum relative error criteria. Nevertheless, they discovered that the exponential model provided the most precise correlation for the density data of binary blends. In another study, Mert et al. [29] produced biodiesel from waste cooking oil and blended it with commercial diesel fuel at various volume percentages. They measured the kinematic viscosity of the prepared blends at the temperature in the range of 273.15 to 373.15 K. Their study demonstrated that a rational model with the functionality of temperature and biodiesel fraction was the most suitable for predicting the kinematic viscosity. They also found that the variation of the kinematic viscosity with biodiesel fraction tends to be linear by increasing the temperature. Mujtaba et al. [30] measured the density and viscosity of individual fuels and ternary biodiesel experimentally within at the temperature of 281.51-348.15 K. They subsequently developed four density and viscosity models to predict the density and viscosity of ternary biodiesel blends at varying blend ratios and temperatures. The results of their stusdies revealed that the exponential regression model was more proficient in estimating the density and viscosity of ternary fuel blends. Salah and Babak [13] estimated the kinematic viscosity of diesel-biodiesel blends using the adaptive neuro-fuzzy inference systems (ANFIS) and least squares support vector machines (LSSVM). They revealed that the predictions of LSSVM with polynomial kernel match very well the experimental data. In other research [16], the viscosity of diesel-biodiesel mixtures was evaluated through four neural network models including the generalized regression neural network, radial basis neural networks, multi-layer perceptron neural network, and cascade

feed-forward neural network. Based on the results of their work, the cascade feed-forward neural network model offers an excellent agreement with experimental data. Jatinder and Ajay [11] examined several neural network architectures, with various training algorithms to predict the different properties, including the flash point, fire point, viscosity, and density, of diesel and biodiesel blends. It was revealed that the neural network having an architecture of 2-7-4 with Levernberg-Marquardt algorithm offers the best approximation for the properties of diesel-biodiesel blends. Ertan and Mustafa [17] blended two different diesel fuels with the biodiesels, produced from six different vegetable oils, and tried to predict the key properties, such as density and viscosities, of the blend by generalized equations and a mixing equation. In their study, for all blends, an excellent agreement between the measured and estimated values of the density and viscosities was observed.

In the present work, the efficiency of NNs and empirical equations, in accurately predicting the kinematic viscosity (KV) and cetane number (CN) of diesel-biodiesel blends at different degrees of blends, are evaluated. The NNs used in this study have different architectures with two or three inputs in which the first and second inputs are constant (biodiesel concentration (%) and the kinematic viscosity (cetane number) of biodiesel). However, the third input parameter is not pre-specified and is selected from among other properties of the biodiesel including the boiling point, evaporation point, flash point, pour point, heat of combustion, cloud point, and specific gravity. So the first goal of this work is to specify a three-input NN with the highest performance and to select the third effective input parameter leading to the best performance of NN. As a second goal, a wide range of two-variable equations will be examined to find the best empirical equations. In this regard, finding the parameters of suggested equations is determined using nonlinear regression. Finally, the best performance of NNs and empirical equation are compared.

## 2. Material and method

### 2.1. Experimental data

In this study, 66 samples of diesel-biodiesel blends consisting of six different methyl esters including soybean methyl ester (SME), canola methyl ester (CME), edible tallow methyl ester (ETME), inedible tallow methyl ester (ITME), low free fatty acid yellow grease methyl ester (LYGME), and high free fatty acid yellow grease methyl ester (HYGME) with varying concentrations (%) (0 to 100) are used to conduct the modeling procedure. These samples are collected from experimental data reported in Kinasts's research [31]. It consists of biodiesel concentration (%) in the diesel-biodiesel blend and the properties of pure biodiesel including the cetane number, boiling point, evaporation point, flash point, pour point, heat of combustion, cloud point, kinematic viscosity, and specific gravity. Also, the kinematic viscosity and cetane number of diesel-biodiesel blends were reported comprehensively.

### 2.2. Neural network models

In this study, a three-layer feed-forward network has been used to predict the KV and CN of diesel-biodiesel blends. In the case of KV, once, the neural network's efficiency is investigated considering two inputs (the concentration (%) of the biodiesel and the KV of biodiesel) and once again it is examined with three inputs. Regarding a three-input NN,

the concentration (%) and KV of the biodiesel are considered as the first and second inputs to the NN, while the third input is selected from the parameters of the cetane number, boiling point, evaporation point, flash point, pour point, heat of combustion, cloud point, and specific gravity. Also, the KV of the blend is defined as the output layer. For the hidden layer, four different transfer functions including log-sigmoid (logsig), tangent-sigmoid (tansig), linear (purelin), and radial basis (radbas) are examined to select the best one. Similarly, the number of neurons in the hidden layer ranges from 1 to 15 to specify an optimum hidden layer size. Finally, the constructed network is trained using four different training algorithms i.e. gradient descent, BFGS quasi-Newton Levenberg-Marquardt, and scaled conjugate gradient. Similar to KV, the CN of the blend is determined through a similar procedure. However, the first input of ANN is the concentration (%) of the biodiesel, the second is the CN of biodiesel, and the last input is selected from other properties of the biodiesel including the kinematic viscosity, boiling point, evaporation point, flash point, pour point, heat of combustion, cloud point, and specific gravity. Also, the CN of the blend is defined as the output layer. In this study, the reported 66 samples are divided randomly with the ratios of 0.7, 0.15, and 0.15, for training, validation, and test respectively. To achieve more precise results, NN is constructed and trained 20 times and the average value is reported. It leads to gaining a reliable performance of the network.

### 2.3. Empirical models

Several two-variable mathematical equations are utilized to predict the KV and CN of the blend. In order to find the best empirical equation and its constants, an unconstrained nonlinear optimization method is applied. The method, which uses the Nelder-Mead simplex algorithm, finds the constants of equations by minimizing the sum of the squares of the differences between the results of the calculation and the experiment. The proper empirical equation describing the relationship between the KV of the blend and two independent variables, i.e. the concentration (%) and the KV of the biodiesel, is detected by checking thirty different empirical equations given in Table 1. In these equations, Z denotes the KV of the blend, and X and Y indicate the concentration (%) and the KV of the biodiesel respectively. Each proposed empirical model is fitted to the existing experimental KV values and compared with each other by determining the mean squared error, correlation coefficient, and coefficient of determination. The same strategy is used to find the best empirical model describing the relationship between the CN of the blend and its variables including the concentration (%) and CN of the biodiesel. In the case of CN and considering equations in Table 1, Z represents the CN of the blend, and X and Y indicate the concentration (%) and CN of the biodiesel respectively.

**Table 1**
List of empirical equations used for describing the KV and CN of the diesel-biodiesel blend.

| Empirical relations | Eq. |
|---|---|
| $Z = A_1X^2 + A_2X + A_3Y^2 + A_4Y + A_5$ | (1) |

$$Z = A_1X^2 + A_2X + A_3Y + A_4 \quad (2)$$

$$Z = A_1X + A_2Y + A_3 \quad (3)$$

$$Z = A_1X + A_2Y^2 + A_3 \quad (4)$$

$$Z = A_1X + A_2Y^2 + A_3Y + A_4 \quad (5)$$

$$Z = A_1X^3 + A_2X^2 + A_3X + A_4Y^2 + A_5Y + A_6 \quad (6)$$

$$Z = A_1X^3 + A_2X^2 + A_3X + A_4Y^3 + A_5Y^2 + A_6Y + A_7 \quad (7)$$

$$Z = A_1 + A_2X + A_3Y + A_4X^2 + A_5XY + A_6Y^2 + A_7X^2Y + A_8XY^2 + A_9Y^3 \quad (8)$$

$$Z = A_1 + A_2X + A_3Y + A_4X^2 + A_5XY + A_6Y^2 + A_7X^2Y + A_8XY^2 \quad (9)$$

$$Z = A_1 + A_2X + A_3Y + A_4X^2 + A_5XY + A_6Y^2 + A_7X^2Y \quad (10)$$

$$Z = A_1 + A_2X + A_3Y + A_4XY + A_5Y^2 \quad (11)$$

$$Z = A_1X^2 + A_2X + A_3Y^2 + A_4Y + A_5XY + A_6 \quad (12)$$

$$Z = A_1Y^3 + A_2Y^2 + A_3Y + \frac{A_4}{(X + A_5)^2} + \frac{A_6}{(X + A_7)} + A_8Y^{A_9}X^{A_{10}} \quad (13)$$

$$Z = A_1Y + \frac{A_2}{(X + A_3)^2} + \frac{A_4}{X + A_5} + A_6Y^{A_7}X^{A_8} \quad (14)$$

$$Z = \frac{A_1Y^{A_2}}{A_3 + A_4X^{A_5}} + A_6\exp(A_7Y + A_8) + A_9\exp(A_{10}X + A_{11}) \quad (15)$$

$$Z = A_1Y^{A_2} + A_3X^{A_4} + A_5 \quad (16)$$

$$Z = A_1\exp(A_2Y + A_3) + A_4\exp(A_5X + A_6) \quad (17)$$

$$Z = \frac{A_1 + A_2Y^{A_3}}{A_4 + A_5X^{A_6}} + A_7\exp(A_8Y + A_9) + A_{10}\exp(A_{11}X + A_{12}) \quad (18)$$

$$Z = A_1\exp(A_2Y + A_3X^2) + A_4\ln(X + A_5) \quad (19)$$

$$Z = \frac{A_1Y^{A_2}}{A_3 + A_4X^{A_5}} \quad (20)$$

$$Z = A_1\exp(A_2Y + A_3) + A_4\exp(A_5X + A_6) \quad (21)$$

$$Z = A_1X^{A_2}Y^{A_3} + A_3 \quad (22)$$

$$Z = A_1X^{A_2} + A_3X^{A_4} + A_5X^{A_6}Y^{A_7} + A_8 \quad (23)$$

$$Z = A_1X^{A_2}Y^{A_3} + A_4 + A_5\exp(A_6X + A_7Y + A_8XY + A_9) \quad (24)$$

$$Z = A_1 + A_2\exp(A_3X + A_4Y + A_5XY + A_6) \quad (25)$$

$$Z = A_1X^{A_2}Y^{A_3}\exp(A_4X + A_5Y + A_6XY + A_7) + A_8 \quad (26)$$

$$Z = A_1X^{A_2}\exp(A_3X + A_4Y + A_5XY + A_6) + A_7 \quad (27)$$

$$Z = A_1 X^{A_2} + A_3 \ln(Y^{A_4}) + A_5 \quad (28)$$

$$Z = A_1 Y^3 + A_2 Y^2 + A_3 Y + A_4 Y^{A_5} X^{A_6} + \frac{A_7}{X + A_8} \quad (29)$$

$$Z = A_1 Y^2 + A_2 Y + A_3 Y^{A_4} X^{A_5} + \frac{A_6}{X + A_7} \quad (30)$$

### 2.4. Performance evaluation

To evaluate the prediction accuracy of artificial neural networks and empirical equations three statistical parameters; the mean square error (MSE), correlation coefficient (R), and coefficient of determination ($R^2$) are used. Eq. 31 and eq. 32 represent formulas for MSE and R parameters respectively. The best model holds the higher R (or R2) and the lower MSE.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Z_{exp,i} - Z_{pred,i})^2 \quad (31)$$

$$R = \frac{\sum_{i=1}^{N}\left(Z_{exp,i} - Z_{exp,mean}\right)\left(Z_{pred,i} - Z_{pred,mean}\right)}{\sqrt{\sum_{i=1}^{N}\left(Z_{exp,i} - Z_{exp,mean}\right)^2}\sqrt{\sum_{i=1}^{N}\left(Z_{pred,i} - Z_{pred,mean}\right)^2}} \quad (32)$$

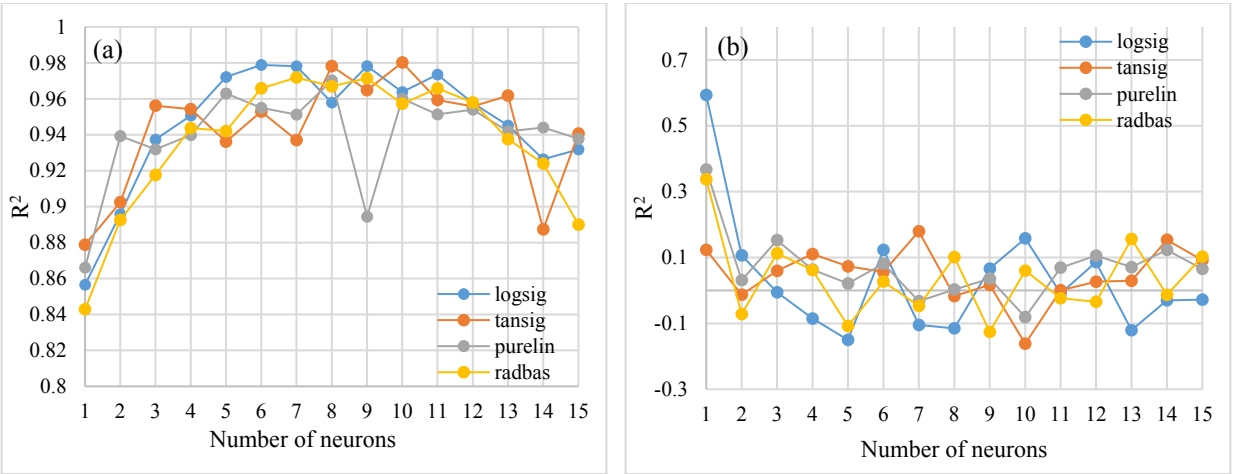
where Z stands for the KV (or CN) of the blend, and "pred" and "exp" subscripts are abbreviations for predicted and experimental values respectively. The subscript "i" stands for the "$i^{th}$" sample and N is the number of samples.

## 3. Results and discussion

### 3.1. ANN results

#### 3.1.1. Cetane number

Figure 1 demonstrates the $R^2$ values for NNs with different combinations of training algorithms and transfer functions. In these NNs, the number of hidden layer neurons is varied from 1 to 15, and the input neuron size is 2. Results show that an ANN model with the Levenberg Marquardt training algorithm, the tansig transfer function, and the hidden neuron size of 10 leads to R and $R^2$-values of 0.9901 and 0.9803 respectively. It has a better performance for predicting the CN of the blend compared to other NNs trained by gradient descent, BFGS quasi-Newton, and scaled conjugate gradient with two inputs.



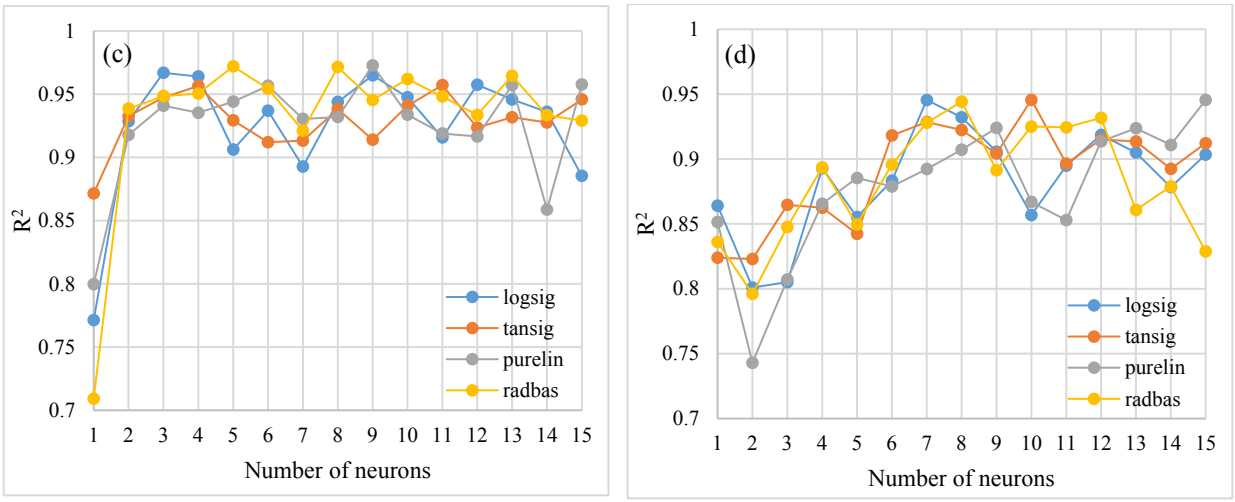



**Figure 1.** $R^2$-values for different combinations of transfer functions, training algorithm, and neuron size with two inputs (a) Levenberg Marquardt, (b) gradient descent, (c) BFGS, and (d) conjugate gradient.

The highest R and $R^2$ and the lowest MSE values for different training algorithms of a two-input ANN are shown in Table 2. As it can be seen, Levenberg Marquardt not only has higher R and $R^2$ values but also possess the minimum MSE value compared to other training algorithms. Thus, it is the best training algorithm for network training.

**Table 2**
Best predicting performance of different training algorithms of a two-input ANN.

| Training func. | Transfer func. | Neuron size | $R^2$ | R | MSE |
|---|---|---|---|---|---|
| Levenberg Marquardt | Tansig | 10 | 0.9803 | 0.9901 | 0.4247 |
| Gradient descent | Logsig | 1 | 0.5932 | 0.7702 | 37.1874 |
| BFGS | Purelin | 9 | 0.9727 | 0.9863 | 0.5355 |
| Scaled conjugate gradient | Tansig | 10 | 0.9456 | 0.9724 | 1.5258 |

Table 3 lists the parameters of the optimum NN model for predicting the CN of the blend with different third inputs. As stated before, the first and second input parameters are constant (i.e. the concentration (%) and CN of the biodiesel) while the third input parameter is chosen by trial-and-error. As shown, the quality of the prediction of an ANN with three input parameters including the concentration (%), CN and cloud point of the biodiesel is far better than that of other ANNs listed in Table 3. In this case, the training algorithm, transfer function, and neuron size are Levenberg Marquardt, purelin, and 4 respectively. Besides, the highest R and $R^2$ values (0.9961 and 0.9922) and lowest MSE value (0.1459) of this ANN depict that it is also more efficient, than the NN with two inputs, in the modeling of the CN of the diesel-biodiesel blend.

### 3.1.2. Kinematic viscosity

Table 4 summarizes the results of different NN architectures with two and three inputs. In all of these networks, the first input parameter is the concentration (%) of the biodiesel and the second one is the KV of the biodiesel. The results show that the third input parameter of the best NN for the present problem is the cloud point. It has 8 neurons with the transfer function of tansig, and has been trained by Levenberg Marquardt having the minimum overall mean square error of 0.0032, and R and

$R^2$-values of 0.9985 and 0.9971. Besides, ANNs with the third input parameter of the boiling point and pour point are the second and third top ANNs describing experimental data very well. Moreover, for all ANNs the Levenberg Marquardt training algorithm is more suitable than other examined training algorithms. It should be noted that the last row in Table 4 is allocated to a NN with 2 inputs. Although the ANN with two inputs also fits very well with existing data, it has lower performance compared to the most of three-input networks.

**Table 3**
Statistical coefficients and parameters of the optimum NNs with different third inputs for predicting cetane number.

| Third input | Training func. | Transfer func. | Neuron size | $R^2$ | R | MSE |
|---|---|---|---|---|---|---|
| Boiling point | Levenberg Marquardt | Purelin | 9 | 0.9656 | 0.9826 | 0.6646 |
| Evaporation point | Levenberg Marquardt | Radbas | 8 | 0.9766 | 0.9882 | 0.5985 |
| Flash point | Levenberg Marquardt | Tansig | 9 | 0.9838 | 0.9919 | 0.3636 |
| Pour point | Levenberg Marquardt | Tansig | 11 | 0.9796 | 0.9897 | 0.5013 |
| Heat of combustion | Levenberg Marquardt | Radbas | 7 | 0.9747 | 0.9873 | 0.5160 |
| Cloud point | Levenberg Marquardt | Purelin | 4 | 0.9922 | 0.9961 | 0.1459 |
| Kinematic viscosity | Levenberg Marquardt | Tansig | 8 | 0.9816 | 0.9908 | 0.4226 |
| Specific gravity | Levenberg Marquardt | Logsig | 8 | 0.9880 | 0.9940 | 0.2726 |

**Table 4**
Statistical coefficients and parameters of the optimum NNs with different third inputs for predicting kinematic viscosity.

| Third input | S | Transfer func. | Neuron size | $R^2$ | R | MSE |
|---|---|---|---|---|---|---|
| Boiling point | Levenberg Marquardt | Radbas | 8 | 0.9965 | 0.9982 | 0.0043 |
| Evaporation point | Levenberg Marquardt | Tansig | 4 | 0.9957 | 0.9978 | 0.0047 |
| Flash point | Levenberg Marquardt | Tansig | 6 | 0.9959 | 0.9979 | 0.0050 |
| Pour point | Levenberg Marquardt | Purelin | 6 | 0.9964 | 0.9982 | 0.0045 |
| Heat of combustion | Levenberg Marquardt | Radbas | 4 | 0.9950 | 0.9975 | 0.0058 |
| Cloud point | Levenberg Marquardt | Tansig | 8 | 0.9971 | 0.9985 | 0.0032 |
| Cetane number | Levenberg Marquardt | Logsig | 5 | 0.9949 | 0.9974 | 0.0059 |
| Specific gravity | Levenberg Marquardt | Radbas | 5 | 0.9947 | 0.9973 | 0.0063 |
| - | Levenberg Marquardt | Purelin | 7 | 0.9957 | 0.9978 | 0.0054 |

### 3.2. Empirical relations

#### 3.2.1. Cetane number

The empirical modeling result indicates the excellent performance of the 18$^{th}$, 14$^{th}$, and 27$^{th}$ empirical equations for describing the CN of the blend. The regression constants for these equations can be observed in Table 5. Also, the statistical coefficients of the top five empirical

equations are summarized in Table 6. As the best predictor model, the 18th empirical model, which is a function of the concentration (%) and CN of the biodiesel, has R, $R^2$ and MSE values of 0.9947, 0.9894, and 0.0959 respectively.

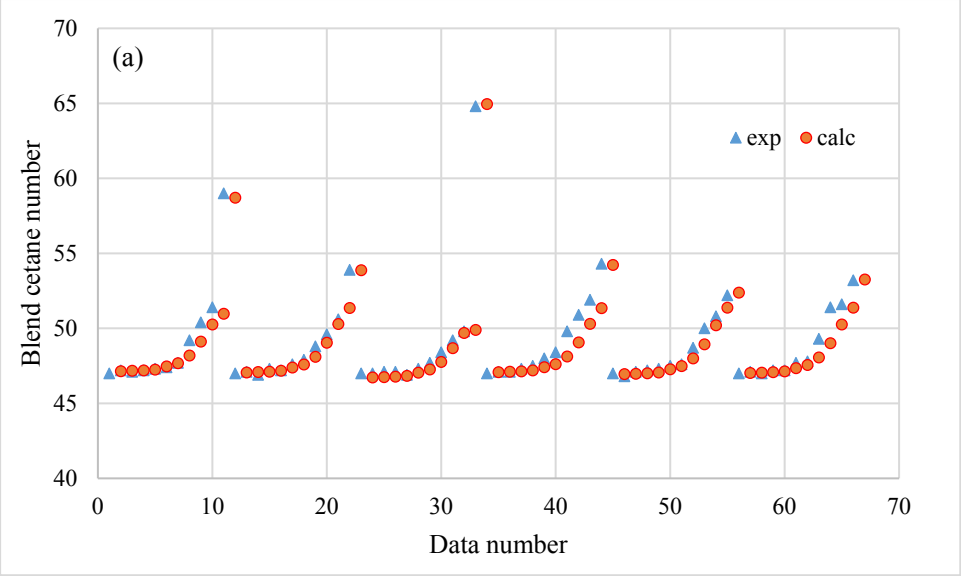
**Table 5**
Regression constants for the top three empirical equations for predicting the CN of the blend.

| Eq. | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ | $A_{10}$ | $A_{11}$ | $A_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 18 | 1046.21 | -0.0023 | 3.26 | 144.89 | $-1.18e^{-5}$ | 3.70 | 8.12 | 0.0084 | 1.50 | -0.0562 | -0.0102 | 5.24 |
| 14 | -0.0277 | -2369797 | 94.95 | 17582 | 56.45 | 7.074 | 6.20 | 22.06 | - | - | - | - |
| 27 | 14468.06 | 0.9368 | -0.1073 | -0.1043 | 0.0019 | -5.9946 | 47.03 | - | - | - | - | - |

**Table 6**
Statistical coefficients of the top five empirical equations for predicting cetane number.

| Eq. | $R^2$ | R | MSE |
|---|---|---|---|
| 18 | 0.9894 | 0.9947 | 0.0959 |
| 14 | 0.9824 | 0.9912 | 0.1594 |
| 27 | 0.9820 | 0.9910 | 0.1622 |
| 29 | 0.9784 | 0.9891 | 0.1956 |
| 30 | 0.9783 | 0.9891 | 0.1963 |

The capability of this empirical model is further supported by Figure 2a and Figure 2b showing the real and predicted cetane numbers. As shown in Figure 2a, the model fits the experimental CN of six different biodiesels with varying concentrations, very well. Also, the illustration of experimental data versus the predictions of the 18th empirical model is given in Figure 2b. Since, the diagonal line in this figure is corresponding to a relative error of zero, the triangular markers aggregation around this line is another robust justification for the excellent performance of the proposed empirical equation.



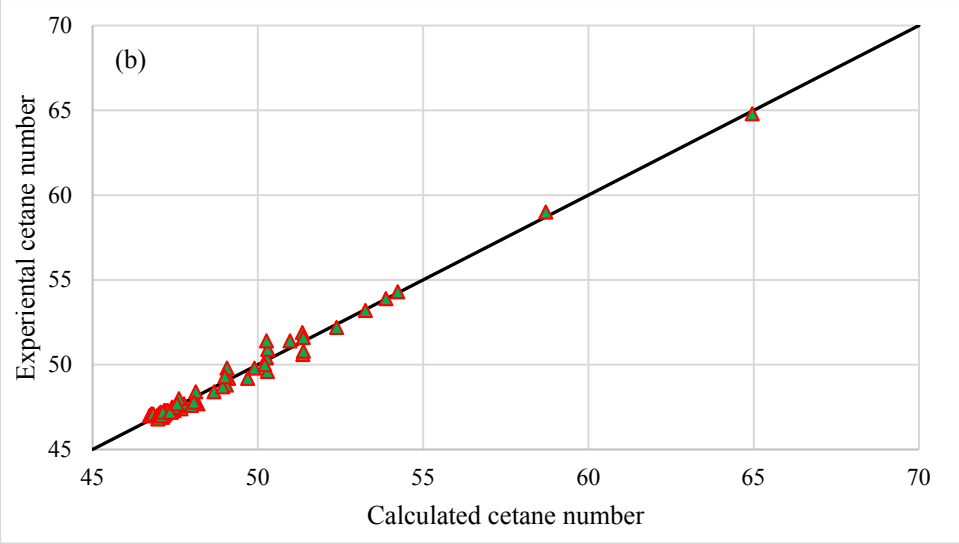



**Figure 2.** Regression plots for the experimental CN and the CN predicted by the 18th empirical equation.

### 3.2.2. Kinematic viscosity

Here, the best empirical equations correlating independent variables (the concentration (%), and KV of the biodiesel) with the KV of the blend are stated. As a result, the 18th, 14th, and 15th empirical equations efficiently estimate the KV of the blend. The regression constants for these equations can be observed in Table 7. Also, Table 8 represents the statistical parameters of the top five empirical relations that sufficiently predict the KV of the blend. It is obvious that the predicted values from the 18th model are in best agreement with the experimental data because of its highest R (0.9980), $R^2$ (0.9960), and lowest MSE (0.0021) values. However, other equations (14, 15, 13, and 29) approximately predict the experimental values as efficiently as the 18th equation. It also seems the 29th equation may be relatively flexible and simple as compared to other equations because it has a simpler form and less regression constant.

**Table 7**

Regression constants for the top three empirical equations for predicting the KV of blend.

| Eq. | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ | $A_{10}$ | $A_{11}$ | $A_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 18 | 0/0444 | -5/9378 | -3/7432 | 0/0215 | -0/0026 | 0/3754 | 14/893 | -0/8990 | 1/3215 | 7837631 | -0/5149 | -16/788 |
| 14 | -0/0002 | -7696/8 | 26/34 | 198/20 | 14/60 | 0/0034 | 1/3525 | 1/0438 | - | - | - | - |
| 15 | 0/0145 | 1/7560 | -0/1062 | 2/0016 | -0/5105 | 0/5573 | 0/0196 | 1/4156 | 0/0007 | 0/4169 | 6/1471 | - |

**Table 8**

Statistical coefficients of the top five empirical equations for predicting kinematic viscosity.

| Eq. | $R^2$ | R | MSE |
|---|---|---|---|
| 18 | 0.9960 | 0.9980 | 0.0021 |
| 14 | 0.9958 | 0.9979 | 0.0022 |
| 15 | 0.9956 | 0.9978 | 0.0023 |
| 13 | 0.9952 | 0.9976 | 0.0026 |
| 29 | 0.9950 | 0.9975 | 0.0027 |

### 3.3. Comparing models

In a comparison between the results obtained from empirical equations and NNs in predicting the CN, the best neural network model with the R-value of 0.9961 offers better prediction than the best empirical equation with the R-value of 0.9947. Also, in the case of KV, the best obtained NN and empirical equation have R-values of 0.9985 and 0.9980 respectively, which indicate that NN has slightly better accuracy, and consequently providing the best agreement between the experimental KV and predicted values of all the examined models in this study.

## 4. Conclusions

In this study, several NNs and empirical models are examined in order to predict the kinematic viscosity of the diesel-biodiesel blend and its cetane number using the experimental data of diesel-biodiesel samples collected from literatures. As a result, for both the kinematic viscosity and cetane number of the diesel-biodiesel blend, the proposed NN model with 3 inputs has higher prediction accuracy than NNs with 2 inputs. The best ANN configuration for predicting the cetane number of the blend comprises three input parameters, namely, the concentration (%), CN and cloud point of the biodiesel. This configuration uses Levenberg Marquardt as the training algorithm, purelin as the transfer function, and four neurons as the neuron size. In this case, the resulting R value is the highest at 0.9961, while the MSE value is the lowest at 0.1459. On the other hand, the best ANN model for estimating the kinematic viscosity of the blend uses eight neurons with tansig as the transfer function and Levenberg Marquardt as the training algorithm. Selecting the concentration (%), KV and cloud point of the biodiesel, in this case, offers the best statistical parameters, i.e., the minimum MSE vlaue of 0.0032, and R value of 0.9985. Furthermore, an empirical equation that consists of exponential and fractional terms holds the highest performance among all the investigated empirical equations in the prediction of either the CN or the KV of blend. In the case of cetane number, the results indicated that when the selected empirical model is a function of the concentration (%) and CN of the biodiesel, the R and MSE values are 0.9947 and 0.0959 respectively. Additionally, the empirical equation that best fits the experimental kinematic viscosity data is a function of the concentration (%) and kinematic viscosity of the biodiesel, with an R value of 0.9980 and an MSE value of 0.0021. This study indicated the high ability of NN and empirical equations in modeling the CN and KV of the diesel-biodiesel blend, however, the accuracy of NNs is higher than empirical equations in terms of R, $R^2$ and MSE values.

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## References

[1] Dharma, S., Haji Hassan, M., Chyuan Ong, H., Hanra Sebayang, A., Susan Silitonga, A., Kusumo, F. and Milano, J., "Experimental study and prediction of the performance and exhaust emissions of mixed *Jatropha curcas-Ceiba pentandra* biodiesel blends in diesel engine using artificial neural networks", *J. Clean. Prod.*, **164**, 618 (2017).

[2] Singh, D., Sharma, D., Soni, S. L., Sharma, S., Kumar Sharma, P. and Jhalani, A., "A review on feedstocks, production processes, and yield for different generations of biodiesel", *Fuel*,

**262**, 116553 (2020).

[3] Athar, M. and Zaidi, S., "A review of the feedstocks, catalysts, and intensification techniques for sustainable biodiesel production", *J. Environ. Chem. Eng.*, **8** (6), 104523 (2020).

[4] Abomohra, A. E. -F., Elsayed, M., Esakkimuthu, S., El-Sheekh, M. and Hanelt, D., "Potential of fat, oil and grease (FOG) for biodiesel production: A critical review on the recent progress and future perspectives", *Prog. Energy Combust. Sci*., **81,** 100868 (2020).

[5] Ramos, M., Dias, A. P. S., Puna, J. F., Gomes, J. and Bordado, J. C., "Biodiesel production processes and sustainable raw materials", *Energies*, **12** (23), 4408 (2019).

[6] Gebremariam, S. and Marchetti, J., "Economics of biodiesel production", *Energy Convers. Manag.*, **168**, 74 (2018).

[7] Alam, M. S. and Tanveer, M. S., Bioreactors, Chapter 5: Conversion of biomass into biofuel: A cutting-edge technology, Elsevier, p. 55 (2020).

[8] Barik, D., Energy from toxic organic waste for heat and power generation, Woodhead Publishing, (2019).

[9] Kassem, Y., Çamur, H. and Bennur, K. E., "Adaptive neuro-fuzzy inference system (ANFIS) and artificial neural network (ANN) for predicting the kinematic viscosity and density of biodiesel-petroleum diesel blends", *Am. J. Comput. Sci. Technol.*, **1** (1), 8 (2018).

[10] Moradi, Gh. R., Mohadesi, M., Karami, B. and Moradi, R., "Using artificial neural network for estimation of density and viscosities of biodiesel–diesel blends", *Chem. Pet. Eng*., **49** (2), 153 (2015).

[11] Kumar, J. and Bansal, A., "Selection of best neural network for estimating properties of diesel-biodiesel blends", *Greece*, **16-19**, 136 (2007).

[12] Gülüm, M., Onay, F. K. and Bilgin, A., "Evaluation of predictive capabilities of regression models and artificial neural networks for density and viscosity measurements of different biodiesel-diesel-vegetable oil ternary blends", *RTU Zinat. Raksti.*, **22** (1), 179 (2018).

[13] Yahya, S. I. and Aghel, B., "Estimation of kinematic viscosity of biodiesel-diesel blends: Comparison among accuracy of intelligent and empirical paradigms", *Renew. Energy*, **177**, 318 (2021).

[14] Piloto-Rodríguez, R., Sánchez-Borroto, Y., Lapuerta, M., Goyos-Pérez, L. and Verhelst, S., "Prediction of the cetane number of biodiesel using artificial neural networks and multiple linear regression", *Energy Convers. Manag.*, **65**, 255 (2013).

[15] Veza, I., Roslan, M. F., Muhamad Said, M. F. and Abdul Latiff, Z., "Cetane index prediction of ABE-diesel blends using empirical and artificial neural network models", *Energy Sources A: Recovery Util. Environ. Eff.*, 1 (2020).

[16] Zheng, Y., Safdari Shadloo, M., Nasiri, H., Maleki, A., Karimipour, A. and Tlili, I., "Prediction of viscosity of biodiesel blends using various artificial model and comparison with empirical correlations", *Renew. Energy*, **153**, 1296 (2020).

[17] Alptekin, E. and Canakci, M., "Determination of the density and the viscosities of biodiesel–diesel fuel blends", *Renew. Energy*, **33** (12), 2623 (2008).

[18] Giakoumis, E. G. and Sarakatsanis, C. K., "Estimation of biodiesel cetane number, density, kinematic viscosity and heating values from its fatty acid weight composition", *Fuel*, **222**, 574 (2018).

[19] Foroughi, B., Shahrouzi, J. R. and Nemati, R., "Detection of gasoline adulteration using modified distillation curves and artificial neural network", *Chem. Eng. Technol.*, **44** (3), 527 (2021).

[20] Babu, D., Thangarasu, V. and Ramanathan, A., "Artificial neural network approach on forecasting diesel engine characteristics fuelled with waste frying oil biodiesel", *App. Energy*, **263**, 114612 (2020).

[21] Raghuvaran, S., Ashok, B., Veluchamy, B. and Ganesh, N., "Evaluation of performance and exhaust emission of CI diesel engine fuel with palm oil biodiesel using an artificial neural network", *Mater. Today: Proc.*, **37**, 1107 (2021).

[22] Humelnicu, C., Ciortan, S. and Amortila, V., "Artificial neural network-based analysis of the tribological behavior of vegetable oil–diesel fuel mixtures", *Lubricants*, **7** (4), 32 (2019).

[23] Kumar, S., Jain, S. and Kumar, H., "Prediction of jatropha-algae biodiesel blend oil yield with the application of artificial neural networks technique", *Energy Sources A: Recovery Util. Environ. Eff.*, **41** (11), 1285 (2019).

[24] Hu, Y. H. and Hwang, J. -N., Handbook of neural network signal processing, Acoustical Society of America, (2002).

[25] Abiodun, O. I., Jantan, A., Omolara, A. E., Dada, K. V., Mohamed, N. A. and Arshad, H., "State-of-the-art in artificial neural network applications: A survey", *Heliyon*, **4** (11), e00938 (2018).

[26] Ghobadian, B., Rahimi, H., Nikbakht, A. M., Najafi, G. and Yusaf, T. F., "Diesel engine performance and exhaust emission analysis using waste cooking biodiesel fuel with an artificial neural network", *Renew. Energy*, **34** (4), 976 (2009).

[27] de Oliveira, F. M., de Carvalho, L. S., Teixeira, L. S. G., Fontes, C. H., Lima, K. M. G., Câmara, A. B. F., Araújo, H. O. M. and Sales, R. V., "Predicting cetane index, flash point, and content sulfur of diesel–biodiesel blend using an artificial neural network model", *Energy & Fuels*, **31** (4), 3913 (2017).

[28] Gülüm, M. and Bilgin, A., "Measurement and prediction of density and viscosity of different diesel-vegetable oil binary blends", *RTU Zinat. Raksti*, **23** (1), 214 (2019).

[29] Gülüm, M., Onay, F. K. and Bilgin, A., "Comparison of viscosity prediction capabilities of regression models and artificial neural networks", *Energy*, **161**, 361 (2018).

[30] Mujtaba, M. A., Kalam, M. A., Masjuki, H. H., Razzaq, L., Khan, H. M., Soudagar, M. E. M., Gul, M., Ahmed, A., Dhana Raju, V., Kumar, R. and Ong, H. C., "Development of empirical correlations for density and viscosity estimation of ternary biodiesel blends", *Renew. Energy*, **179**, 1447 (2021).

[31] Kinast, J. A., Production of biodiesels from multiple feedstocks and properties of biodiesels and biodiesel/diesel blends, National Renewable Energy Laboratory, Battelle, (2003).