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Fuzzy Logic-based Gain Scheduling for PI control of a Nonlinear pH Process

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ABSTRACT

pH control is crucial in biological and water treatment processes, yet it poses significant challenges due to its nonlinear characteristics. Conventional Proportional-Integral-Derivative (PID) controllers, although operator-friendly, often struggle to maintain satisfactory performance in highly nonlinear systems, especially when subjected to disturbances, measurement delay, and measurement noise. On the other hand, gain scheduling is a technique to cope with the process nonlinearity while maintaining the PID simplicity for the operators. In this paper, a gain-scheduled digital proportional-integral (PI) controller is presented for pH control, where the parameters of the controller are adapted using fuzzy logic. The control error and its numerical derivative are fed to the fuzzy inference unit with 7 membership functions, fuzzifying the severity of the control deviation. Then, the PI proportional and integral gains are updated in the defuzzification step. A simulation of a benchmark pH process with measurement delay was carried out under various scenarios. The results show that the proposed fuzzy-PI controller significantly outperforms the conventional PI controller. This is especially true in cases of feed disturbance and considerable changes in the setpoint, where the nonlinear process deviates significantly from the nominal point. In a combined disturbance/setpoint change scenario, the fuzzy-PI controller reduces the integral of the absolute error by 54%.

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1. Introduction

The control of pH is of critical importance in biological and wastewater treatment systems. In bio-processes, pH variations can lead to cell lysis and significantly affect the overall performance of the system [1]. However, pH control poses significant challenges due to its inherent nonlinearity that stems from a

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complicated relation between pH and concentrations, as reflected in the titration curve [2,3]. Proportional-Integral (PI) controllers are arguably the most popular feedback controllers in the industry because of their simplicity and efficiency [4]. Nevertheless, their performance is often found to be inadequate for highly nonlinear processes such as pH control [5]. The inherent nonlinearity and high sensitivity of pH processes give rise to a challenging control problem [6].

A well-known strategy to counter the process nonlinearity is the use of nonlinear, adaptive control approaches [7], such as those based on Lyapunov stability [8]. There is a considerable research on the adaptive control of pH processes. For example, an indirect adaptive controller was developed in [9], which employed a backstepping technique combined with a parameter estimator to deal with parametric uncertainties in pH control. An adaptive control approach utilizing a fuzzy approximator for unknown model nonlinearities was proposed in [10]. A multiple model approach was developed in [11] in order to handle the nonlinearity of the titration curve and its variations during pH control. Later, the fuzzy modeling of the titration curve for the adaptive pH control was presented in [12]. Although adaptive control can be effective in dealing with the process nonlinearity, it often requires a process model and considerable effort to derive the control law. Also, the assumptions made during control law derivation limit the practicality of the results and make them vulnerable to model mismatch. Moreover, the resulting control law generally deviates from standard industrial forms and, thus, is hard for plant operators to maintain.

Model Predictive Control (MPC) has been applied to pH control extensively (see e.g., [13–16]. As an advanced control strategy, MPC is able to cope with the process nonlinearity and handle process constraints systematically. However, MPC requires a relatively accurate dynamic prediction model and high expertise level. Coupled with its computational burden, these factors limit its practicality for routine pH processes.

Fuzzy logic-based controllers represent an effective approach to handling nonlinear and uncertain systems [17], particularly in biological wastewater treatment processes [18]. Fuzzy-based controllers rely on a set of qualitative rules and human-like linguistic judgements to infer an appropriate numerical action for the manipulated variable [19].

A rich literature exists on the application of fuzzy logic in chemical process control. Some examples include fuzzy controllers for the tank liquid level [20], heat exchanger temperature [21], pressure control [22], and pH in a wastewater treatment plant [23]. Fuzzy logic can be used to calculate the control action directly, resulting in a stand-alone fuzzy controller [24]. A drawback of this scheme is that the resulting controller is difficult to interpret or maintain by regular plant operators, who are mostly familiar with proportional-integral-derivative (PID) controllers. Consequently, the operators are generally not expected to know the underlying fuzzy principles and be able to troubleshoot a fuzzy controller when necessary. Fuzzy logic can also be used in conjunction with conventional PID controllers, where the PID parameters are adjusted in real time by fuzzy logic in order to adapt to the process nonlinearity (see e.g., [22,25]). The latter falls within the category of gain scheduling PID controllers [24].

Gain scheduling offers an operationally simple way to cope with process nonlinearities as it preserves the well-established PID structure. It has been applied to a variety of control problems, including temperature control in continuous stirred tank reactors [26] and fluid catalytic cracking [27], and concentration control in wastewater treatment [28]. Regarding the pH process, there are several implementations of gain scheduling PID in the literature. A gain scheduling scheme for pH control was presented in [29], where the proportional gain of the controller was adjusted based on the gain of the current process derived from the titration curve. In another study, a fuzzy logic scheme for online tuning of PI parameters in pH control was presented [30]. In that scheme, fuzzy logic uses the control error and its numerical derivative as the inputs. It then calculated a single adaptation parameter that would adjust both proportional and integral gains. Therefore, the controller is a fuzzy self-tuning PI controller with only one degree of freedom. The reason is that both proportional gain and integral time are guided by a single parameter, which is updated by the fuzzy adaptation mechanism. Also, the performance tests were presented for disturbance scenarios only, and the setpoint tracking performance was not presented. Fuzzy logic was also used in the gain scheduling control of pH in [31]. Similar to [29], the gain scheduling scheme was based on the nonlinear titration curve. Specifically, the titration curve was split into a number of regions, and a set of controller gains was tuned for moving from one region to another. Then, a Sugeno-type fuzzy system was employed to smooth out the sharp gain variations using the membership functions defined for different regions of the titration curve. Therefore, the inputs to fuzzy logic were the pH setpoint and its current value, which would represent the current point on the titration curve. Although the titration curve captures most of the process nonlinearity, its curvature and slope are subject to change when disturbances, in such parameters, as the feed concentrations, occur. Therefore, a gain scheduling based on a single (nominal) titration curve may not provide adequate control. Recently, fuzzy control was used in combination with a fixed-parameter PID, where PID and the fuzzy controller were used for small and large control deviations, respectively [32]. Since the PID parameters were fixed, the method proposed in [32] would not be considered fuzzy gain scheduling.

Motivated by the operational simplicity of PI controllers and the ability of fuzzy logic to adapt to the process nonlinearity, a fuzzy gain scheduling digital PI (called fuzzy-PI) controller is developed in this work. The fuzzy scheduler allows for the independent tuning of the proportional and integral gains. In contrast to [30], the proposed fuzzy-PI controller offers two degrees of freedom for the online update of the proportional and integral gains. Evidently, a two-parameter controller is more flexible as the underlying fuzzy logic can adjust the proportional and integral gains separately to meet the needs of the closed-loop system. In this way, the controller speed can be enhanced while avoiding an excessively oscillatory or an unstable closed loop. The proposed fuzzy-PI takes the control error and its numerical derivative as the inputs and employs a Mamdani-type fuzzy system to obtain the appropriate values of the gains of the controller dynamically. The proposed controller requires no process model. It is also easy to use owing to the simple PI structure, and is shown to outperform a tuned conventional PI, especially in disturbance scenarios and in the presence of measurement delays.

The remainder of the paper is organized as follows. Section 2 presents the dynamic model of the pH process, which is followed by a description of the fuzzy-PI controller in Section 3 and simulation studies in Section 4. Finally, concluding remarks are provided in Section 5.

2. Process Model

As shown in Figure 1, the process involves HNO_3 as a strong acid (q_1), NaOH as the titrant base (q_2), and $NaHCO_3$ as the buffer (q_3). The control valve is placed on the base flow as the manipulated variable. The process model is taken from [33]. The modeling assumptions include perfect mixing, constant density, constant volume, and complete solubility of the ions. The following chemical reactions take place [33].

$$H_2CO_3 \Leftrightarrow H^+ + HCO_3^- , \tag{1}$$

$$HCO_3^- \Leftrightarrow H^+ + CO_3^{2-} , \tag{2}$$

$$H_2O \Leftrightarrow H^+ + OH^- , \tag{3}$$

with the dissociation constants given below.

$$K_{a_1} = \frac{[H^+][HCO_3^-]}{[H_2CO_3]} , \tag{4}$$

$$K_{a_2} = \frac{[H^+][CO_3^{2-}]}{[HCO_3^-]} , \tag{5}$$

Assuming the concentration of water is constant ($[H_2O] \approx 1$

$$K_w = \frac{[H^+][OH^-]}{[H_2O]} = [H^+][OH^-]. \tag{6}$$

Using the concept of reaction invariants [34], the following variables are defined.

$$W_{ai} = [H^+]_i - [OH^-]_i - [HCO_3^-]_i - 2[CO_3^{2-}]_i , \tag{7}$$

$$W_{bi} = [H_2CO_3^-]_i + [HCO_3^-]_i + [CO_3^-]_i \ ,(i = 1,2,3,4). \tag{8}$$

The mmass balances for the invariants around the tank yield:

$$V\frac{dW_{a_4}}{dt} = (W_{a1} - W_{a4})q_1 + (W_{a2} - W_{a4})q_2 + (W_{a3} - W_{a4})q_3 , \tag{9}$$

$$V\frac{dW_{b_4}}{dt} = (W_{b1} - W_{b4})q_1 + (W_{b2} - W_{b4})q_2 + (W_{b3} - W_{b4})q_3 , \tag{10}$$

The outlet pH value can be obtained from the following algebraic equation [9,33].

$$W_{a4} + 10^{pH-14} - 10^{-pH} + W_{b4}\frac{1 + 2 \times 10^{pH-pK_2}}{1 + 10^{-pH+pK_1} + 10^{pH-pK_2}} = 0 , \tag{11}$$

$$pK_1 = -\log_{10} K_{a1} \ ,pK_2 = -\log_{10} K_{a2}. \tag{12}$$

The nominal operating conditions of the process are given in the Appendix.

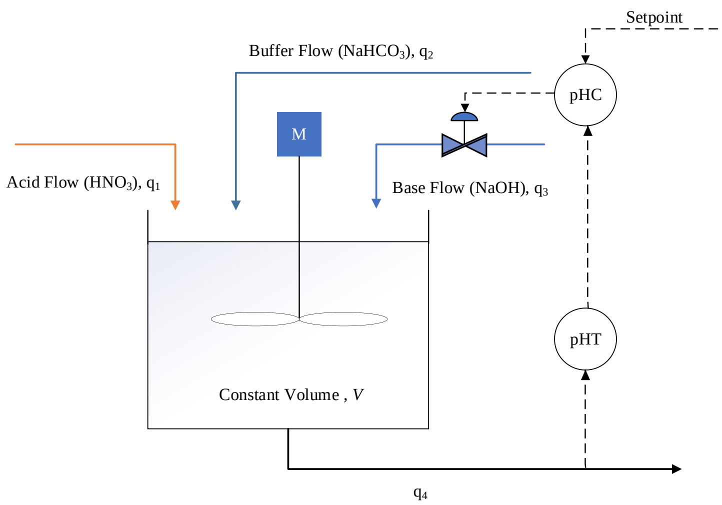

Figure 1. Schematic diagram of the pH process.

3. Proposed Fuzzy-PI Controller

The high nonlinearity of the pH process is apparent from Eq. (11). The gain of the process is subject to large changes for a nonlinear process, undermining the performance of linear controllers such as PI with constant tuning parameters. In extreme cases, severe oscillations or even instability may occur. For this reason, a fuzzy-PI controller has been developed for this process, in which the PI parameters are varied online based on fuzzy logic, as shown in Figure 2. In this way, the operator-friendliness and simplicity of PI controllers are merged with the capabilities of fuzzy logic in dealing with nonlinear behaviors. The PI equation is written in discrete form as [35]:

$$\begin{aligned} p(k) &= p(k-1) + K_P \Delta e(k) \\ &\quad + K_I e(k)\Delta t \\ \Delta e(k) &= e(k) - e(k-1), \end{aligned} \tag{13}$$

where k denotes the current sample, Δt is the sample time, $p(k)$ is the current controller output, $e(k)$ is the current control error, and K_P and K_I are respectively the proportional and integral parameters of the controller.

The fuzzy logic receives the control error (e) and its derivative (Δe) to calculate appropriate values for K_P and K_I [22,36]. Before being processed by the fuzzy inference system (as shown in Figure 3), the input error and its derivative are fuzzified and transformed into linguistic values.

In this context, seven triangular membership functions are used for fuzzification (i.e., "NB" for negative big, "NM" for negative medium, "NS" for negative small, "Z" for zero, "PS" for positive small, "PM" for positive medium, and "PB" for positive big). After the fuzzification stage, using a rule base adapted from [36,37], Proportional and integral gains are generated, as provided in Tables 1 and 2. For defuzzification and obtaining the final values of K_P and K_I according to Figures 4 and 5, seven triangular membership functions are used (i.e., "VS" for very small, "SM" for small medium, "S" for small, "M" for medium, "B" for big, "BM" for big medium, and "VB" for very big). The defuzzification lower limits are set to zero to enable zero gain if needed. The upper limits are obtained from running a series of servo and regulatory experiments. The objective is to arrive at a limit that is neither too tight to restrict the gain variability, nor too wide to risk instability. The ultimate gain estimated by the relay auto-tuning method (Section 4) can serve as a guide. However, due to the process nonlinearity, the ultimate gain is not constant throughout the operation. Therefore, fine tuning is also performed to find appropriate limits. A sensitivity analysis on the chosen upper limits is conducted in Section 4.1.1.

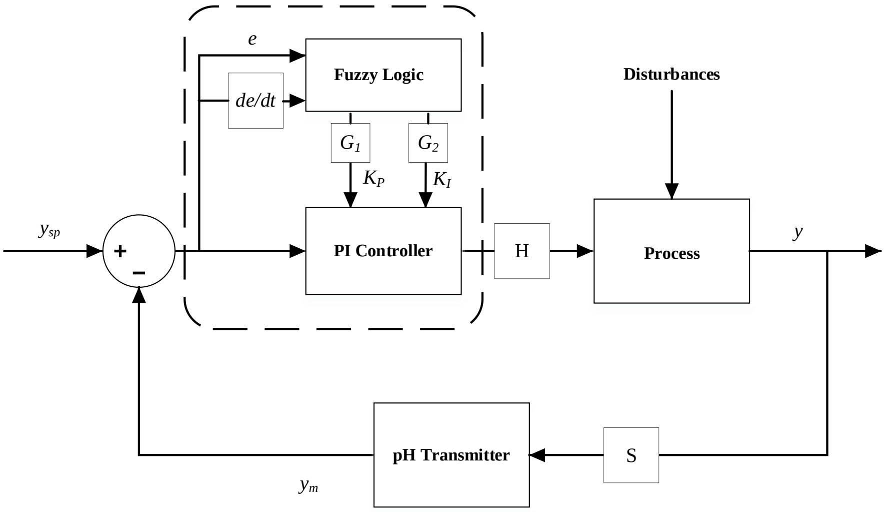

Figure 2. Block diagram of the process with its fuzzy-PI controller. The blocks H and S indicate zero-order hold and sample, respectively.

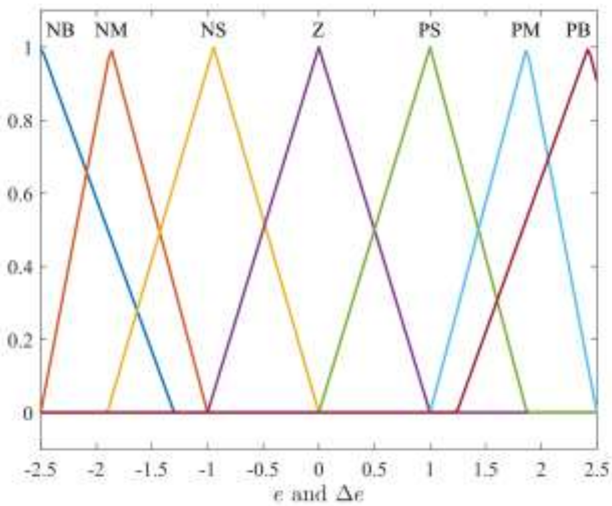

Figure 3. Triangular membership function of e and ***Δ***e.

A typical rule similar to the following is illustrated.

If e is NB and Δe is NB, then K_P is VB and K_I is VS

Table 1.
Fuzzy Control Rules for K_P

		Δe						
		NB	NM	NS	Z	PS	PM	PB
	NB	VB	VB	BM	BM	B	B	M
	NM	VB	VB	BM	BM	B	M	S
	NS	BM	BM	BM	B	M	S	SM
e	Z	BM	B	B	M	S	SM	SM
	PS	B	PS	M	S	S	SM	SM
	PM	B	M	S	SM	SM	SM	VS
	PB	M	S	S	SM	SM	VS	VS

Table 2.
Fuzzy Control Rules for K_I

		Δe						
		NB	NM	NS	Z	PS	PM	PB
	NB	VS	VS	VS	SM	SM	S	M
	NM	VS	VS	SM	SM	S	M	B
	NS	SM	SM	S	S	M	B	B
e	Z	SM	S	S	M	B	B	BM
	PS	S	S	M	B	B	BM	BM
	PM	S	M	B	BM	BM	VB	VB
	PB	M	S	B	SM	VB	VB	VB

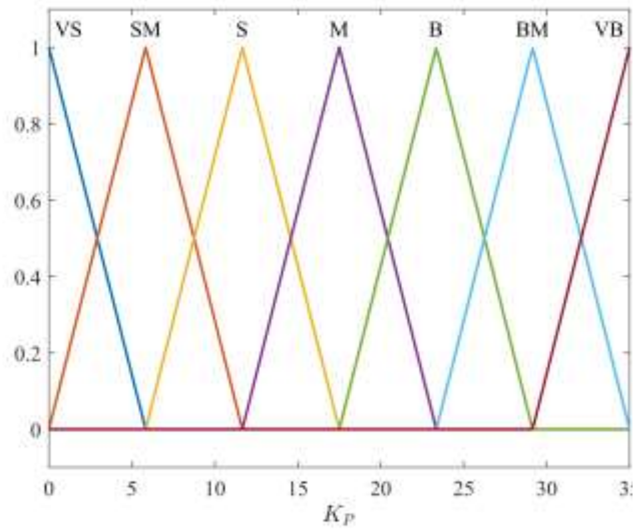

Figure 4. Triangular membership functions for K_P.

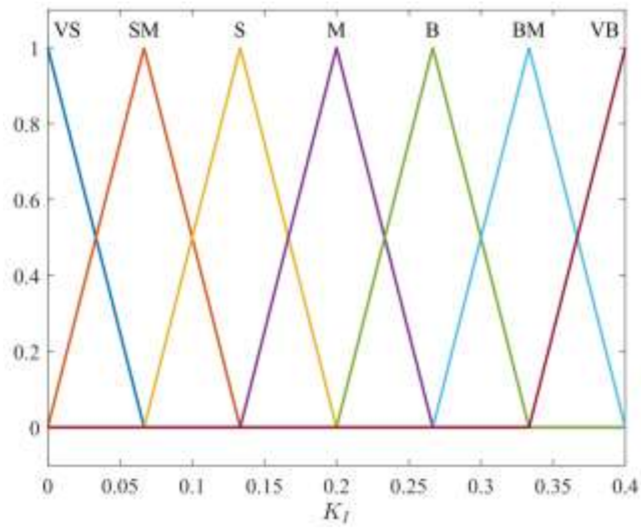

Figure 5. Triangular membership functions for K_I.

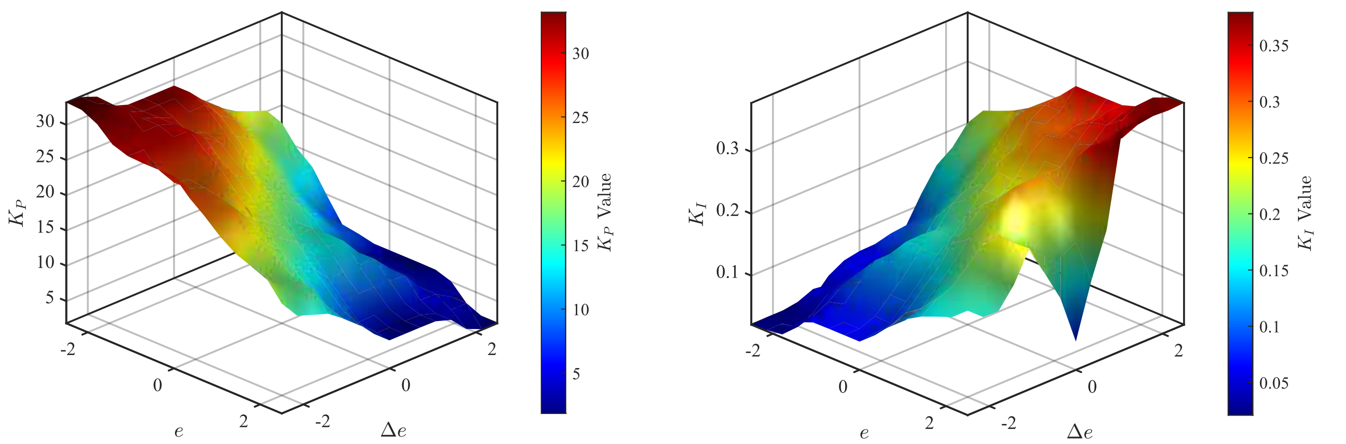

Figure 6. Fuzzy surfaces for the parameters of the controller.

The fuzzy inference system is utilized to determine the parameters of the controller according to the value of the control error (e) and its derivative (Δe). The fuzzy surfaces corresponding to the defuzzification step are illustrated in Figure 6. These fuzzy surfaces provide real-time gain adjustment in response to process dynamics. The output of the fuzzy surfaces is nonsmooth in general. Therefore, the parameters of the controller calculated by fuzzy logic may experience sharp changes, which could lead to undesired fluctuations. To avoid this issue, the defuzzification outputs are passed through a discrete filter in order to apply a weighted average of the previous and the current outputs [30,38]:

$$\overline{K_P}(k) = qK_P(k-1) + (1-q)K_P(k)\ , \tag{14}$$

$$\overline{K_I}(k) = qK_I(k-1) + (1-q)K_I(k), \tag{15}$$

where $q \in (0,1)$ is the damping coefficient, and the accent $\overline{(.)}$ indicates the values smoothed out by the filter and sent to the PI controller. The damping coefficient balances between smoothness and responsiveness of the filter output (here the controller gains). The closer it is to zero, the faster the changes are. However, this could lead to aggressive control actions. A damping coefficient close to 1 yields smoother variations. However, the responsiveness can be compromised, leading to slow closed-loop response.

4. Results and Discussions

In this section, the performance of the conventional and the fuzzy-PI controllers is compared for a variety of setpoint tracking, disturbance rejection, and measurement noise scenarios. The simulations are performed in MATLAB/Simulink. The fixed-step solver ODE5 (Dormand-Prince) with a time step of 0.01 s and a sample time of 2 s is used to run the simulations. It is noted that the control algorithm is implemented in a digital manner (i.e., using sample and zero-hold). Therefore, a fixed-step solver must be used for integrating the simulation model. To ensure the solution accuracy, other fixed-step solvers such as ode3 (Bogacki-Shampine) and ode 1be (backward Euler) with smaller step sizes were also attempted. However, the difference was negligible. The simulated equations include the process model (9-12), the PI controller (13), the fuzzy system, and the filter equations (14-15). The damping coefficient in Eqs. (14-15) was set to 0.3 to balance between smoothness and responsiveness of the control action. Preliminary tests confirmed that the closed-loop response was not sensitive to values within +40% of the chosen value. However, smaller values would lead to undesired fluctuations in the gain values, especially in the presence of noise.

A measurement delay of two sample times (4 s) is assumed for the pH transmitter to reflect sensor limitations. It is evident from control theory that excessively large delay values cause control degradation and lead to instability for any feedback controller. Therefore, a study on the effect of delay on the control performance is outside the scope of this work. Interested readers are referred to control textbooks [35] for delay handling techniques.

The fixed-parameter PI is tuned using the well-established relay auto-tuning method proposed in [39]. In this method, the closed-loop system is put into small, uniform oscillations using an on-off controller with a certain deadband (see Figure 7). This approach has gained widespread industrial use and implemented in commercial simulators such as Aspen HYSYS® and Unisim Design® owing to the following advantages: it is model free; it requires only a single closed-loop experiment;

it avoids forcing the process to its stability limit; and it allows the oscillation magnitude to be safely adjusted.

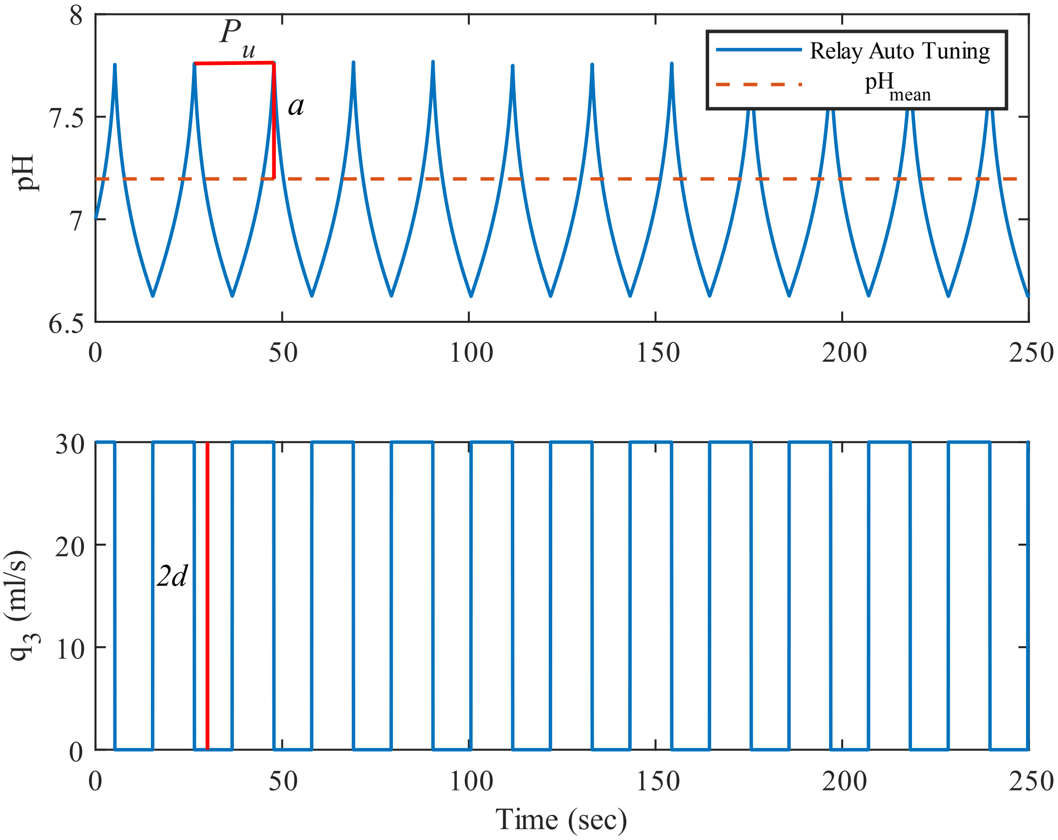

Figure 7. Auto-tuning of the conventional PI using a relay controller.

The relay amplitude, d, was set to 15 ml/s, corresponding to a ±15 ml/s deviation around the nominal base flow of 15.8 ml/s (Table A1). The amplitude of the oscillation of the controlled variable, a, was determined as the vertical distance from the mean pH value to the peak oscillation (a=0.57). As shown in Figure 7, the relay test yields an ultimate period of P_u=21.3 sec. The ultimate gain is then approximated as $K_{cu} = \frac{4d}{\pi a} = 33.5$. Finally, the tuning values are read from the Ziegler-Nichols table [35] as $K_P = 15.08$ and $K_I = 0.85$. However, these tunings were found to be too aggressive for the nonlinear pH process. Therefore, fine-tuning resulted in the proportional and integral coefficients of $K_P = 10$ and $K_I = 0.1$, respectively. This improved the performance of the conventional PI controller in the presence of measurement delays and disturbances, as presented in the simulation studies.

4.1. Setpoint tracking

In the first scenario, the pH setpoint is changed from 7 to 7.5. As shown in Figure 8, both fuzzy-PI and conventional PI controllers perform satisfactorily. The conventional PI is even less oscillatory.

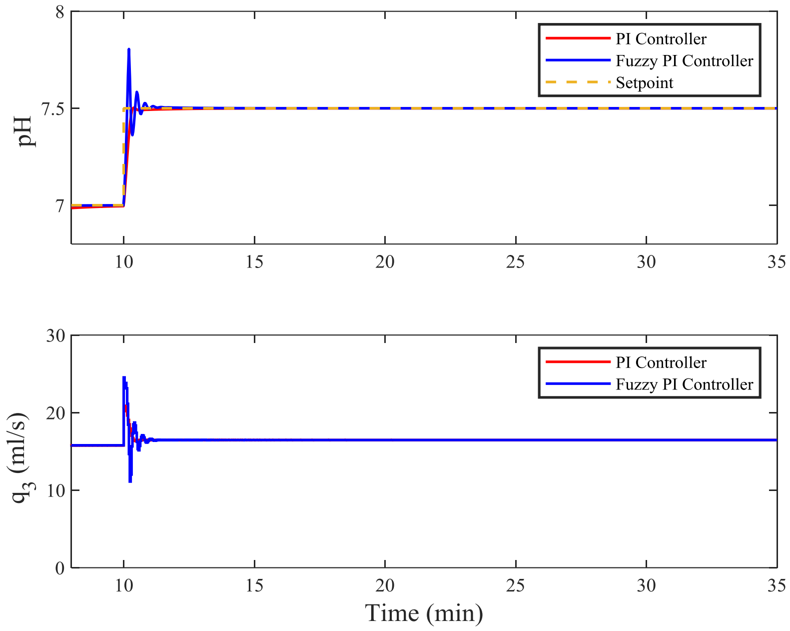

Figure 8. Comparing conventional and fuzzy-PI controllers in Scenario 1.

In the Scenario 2, the pH setpoint is reduced from 7 to 5.5. As shown in Figure 9, both controllers exhibit satisfactory performance. However, the fuzzy-PI controller reaches the new setpoint slightly faster. Both controllers hit the lower saturation limit for a short time. To investigate the effect of reset windup on the control performance, the simulations were rerun with anti-reset windup enabled. This works by stopping the integral action while the manipulated variable is saturated. The results showed no considerable difference compared with Figure 9. The reason is that the saturation occurs for a very short time, and thus, the effect of reset windup is minimal.

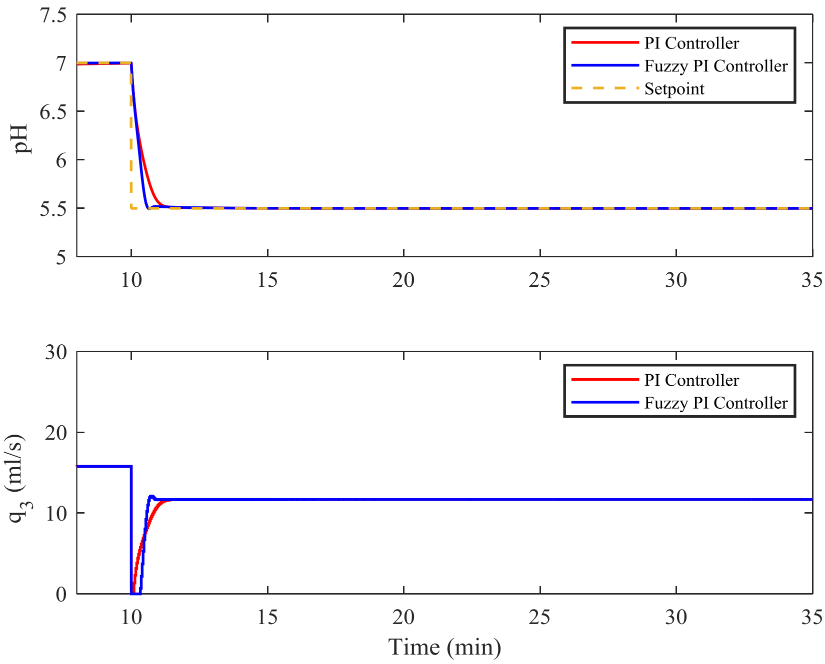

Figure 9. Comparing conventional and fuzzy-PI controllers in Scenario 2.

4.1.1 Sensitivity analysis of defuzzification ranges

To assess the sensitivity of the fuzzy scheduler to the defuzzification ranges of K_P and K_I, two additional tests are conducted in Scenario 1. In the first test, the defuzzification upper limit for both controller gains is increased by 15%. As shown in Figure 10, a higher upper limit produces a more aggressive response. Specifically, the rise time decreases and noticeable oscillations are introduced.

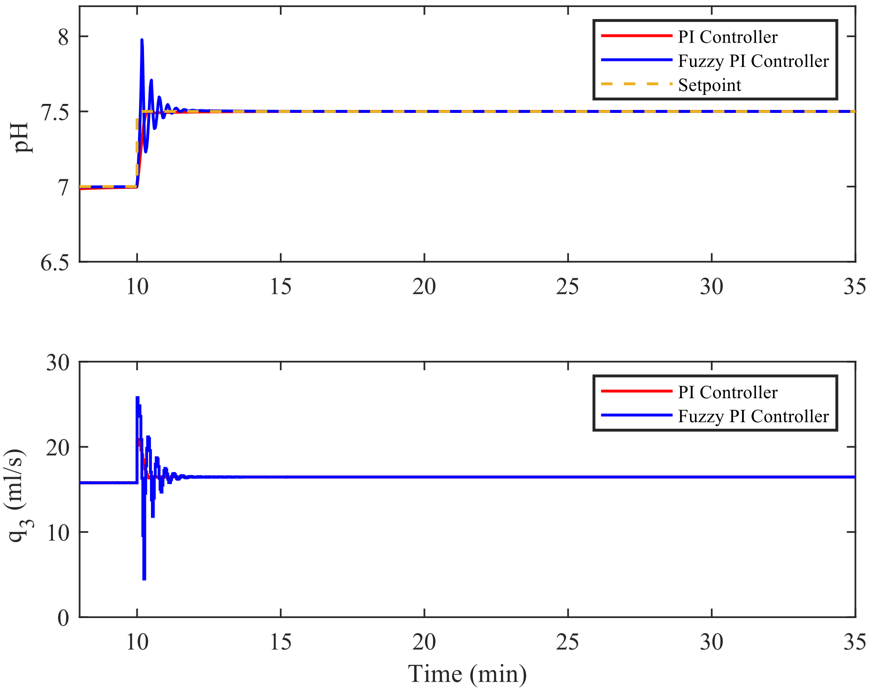

Figure 10. Performance of the fuzzy-PI controller in Scenario 1 with a 15% increase in the K_P and K_I defuzzification upper limit.

In the second test, the upper limits are decreased by 15%. As illustrated in Figure 11, this leads to a more conservative control response with improved damping. These sensitivity results confirm that the original upper bounds (Figures 4 and 5) deliver satisfactory performance. Moreover, while the fuzzy PI is shown to be robust to moderate variations in the defuzzification upper bounds, a reduced upper bound yields a less oscillatory response.

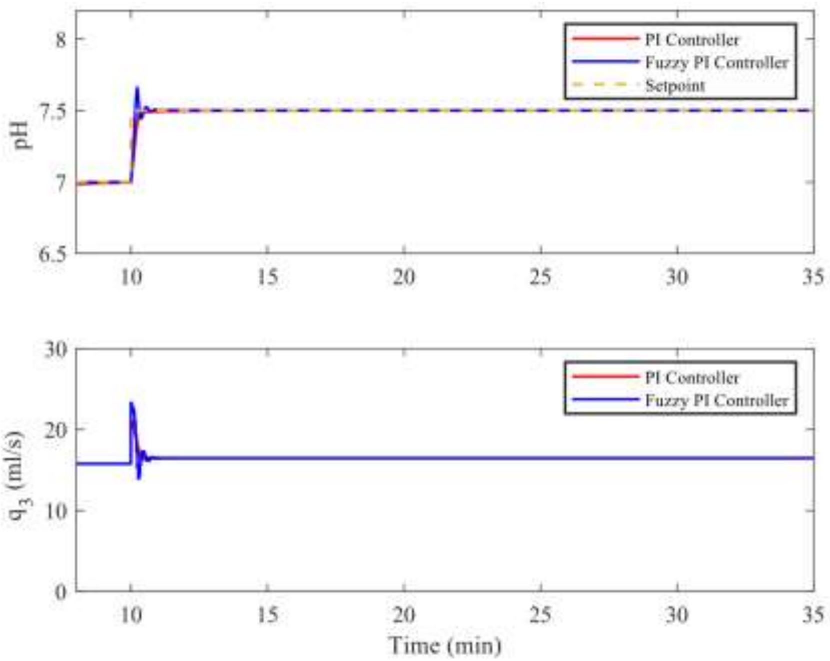

Figure 11. Performance of the fuzzy-PI controller in Scenario 1 with a 15% decrease in the K_P and K_I defuzzification upper limit.

4.2. Disturbance rejection

In Scenario 3, simultaneous disturbances in the buffer and acid flow rates and the reaction invariants of the acid stream are considered. See Figures 12-13 for the disturbance trajectories and Table A1 for nominal values of the reaction invariants.

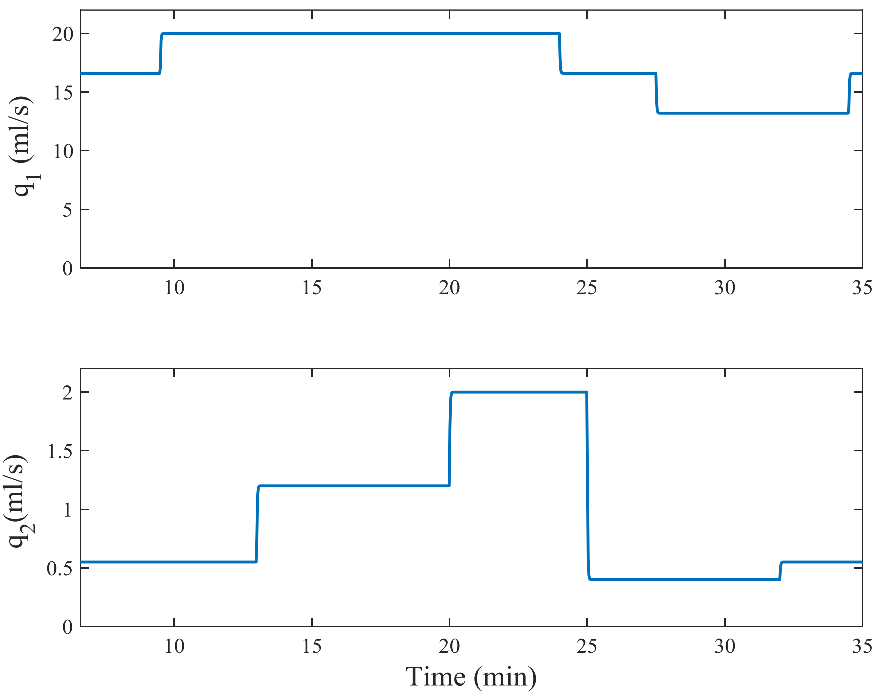

Figure 12. Disturbances in the acid and buffer flow rates in Scenario 3.

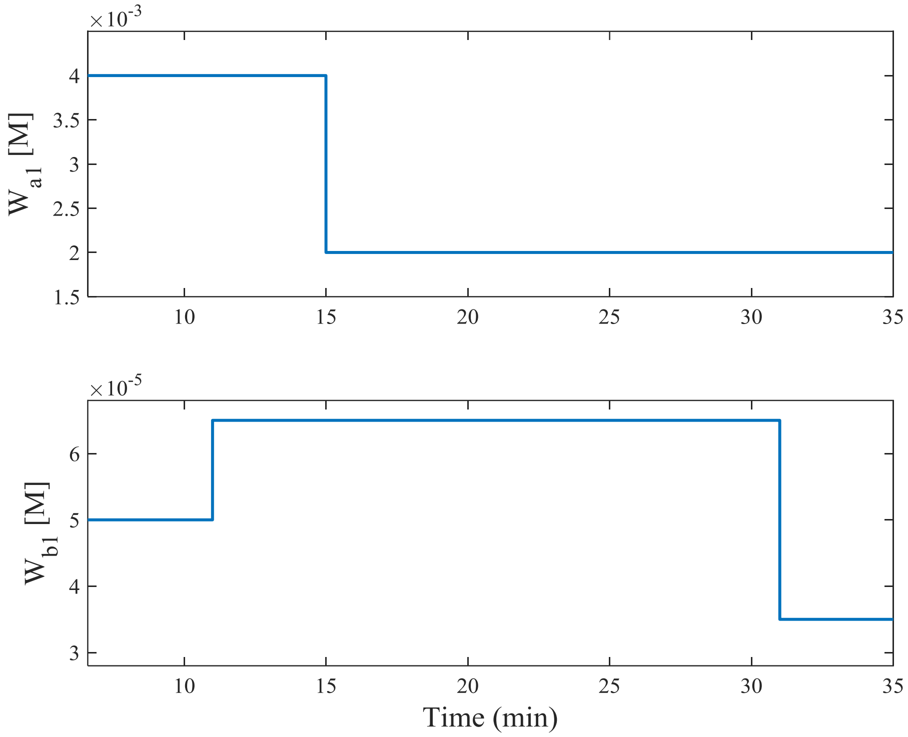

Figure 13. Disturbances in the reaction invariants of the acid stream in Scenario 3.

The control performance in the disturbance rejection scenario 3 is depicted in Figure 14. Despite the varying feed reaction invariants, the fuzzy-PI controller is able to bring the pH back and maintain it close to the setpoint after each disturbance step. On the other hand, the conventional PI experiences larger deviations with a large overshoot after the t=15 min.

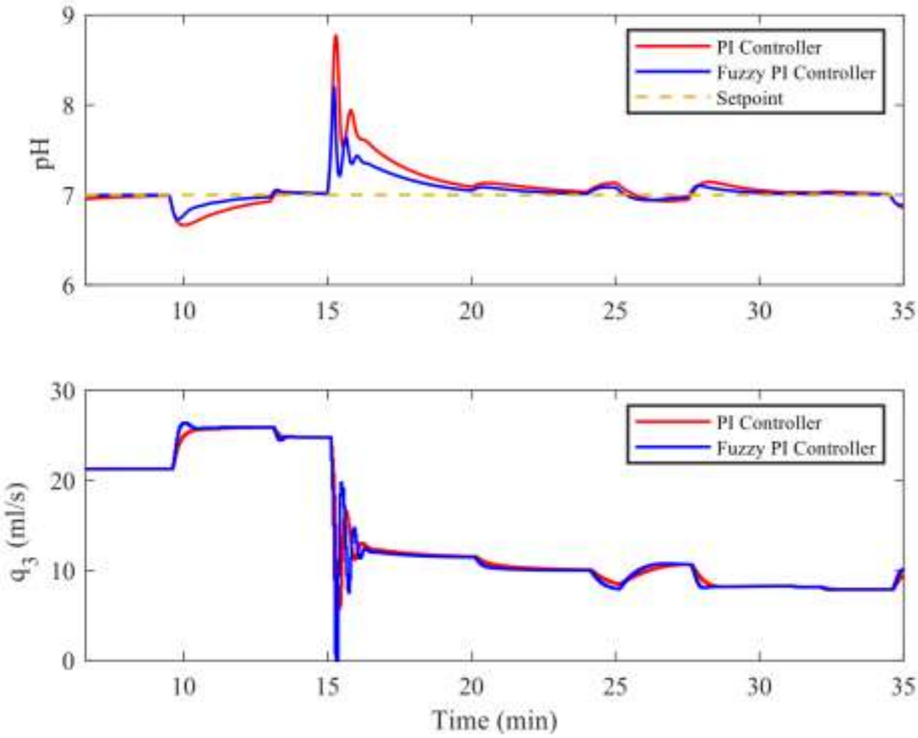

Figure 14. Comparing conventional and fuzzy-PI controllers in Scenario 3.

The time variations of the fuzzy-PI parameters in Scenario 3 are shown in Figure 15. It is seen that the fuzzy logic is able to adjust the proportional and integral gains dynamically as soon as the pH deviates from the setpoint until a steady state is reached. These variations yield appropriate control actions that are neither too weak nor too aggressive, thereby ensuring sufficiently fast, yet stable performance.

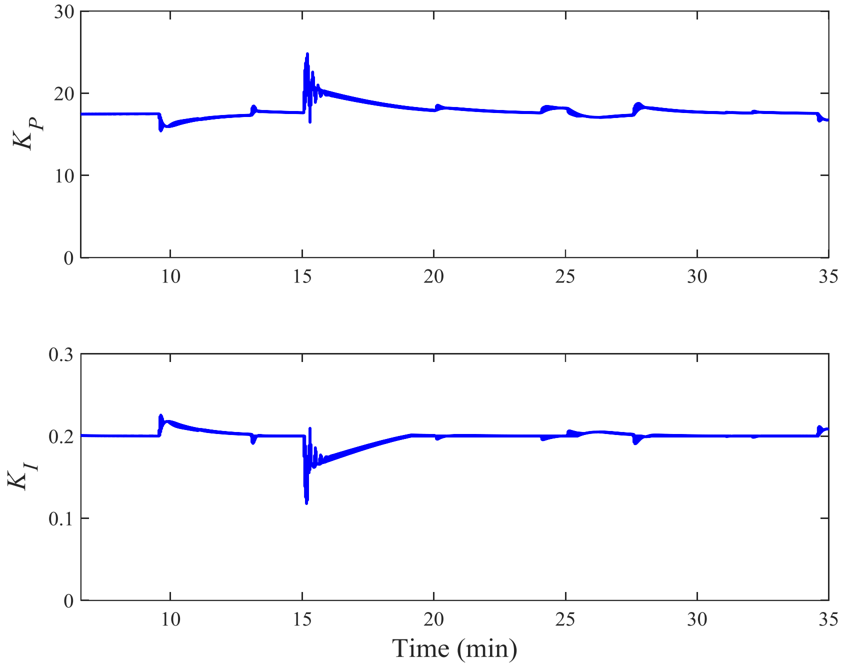

Figure 15. Proportional and integral gains varied by fuzzy logic in Scenario 3.

4.3. Combined setpoint tracking and disturbance rejection

In Scenario 4, disturbances and setpoint changes are applied concurrently. Specifically, concentration disturbances occur in the acid stream at t=9, 19, and 28 min, as shown in Figure 16. Also, the pH setpoint is decreased from 7 to 5.5 at the t=8.1 min.

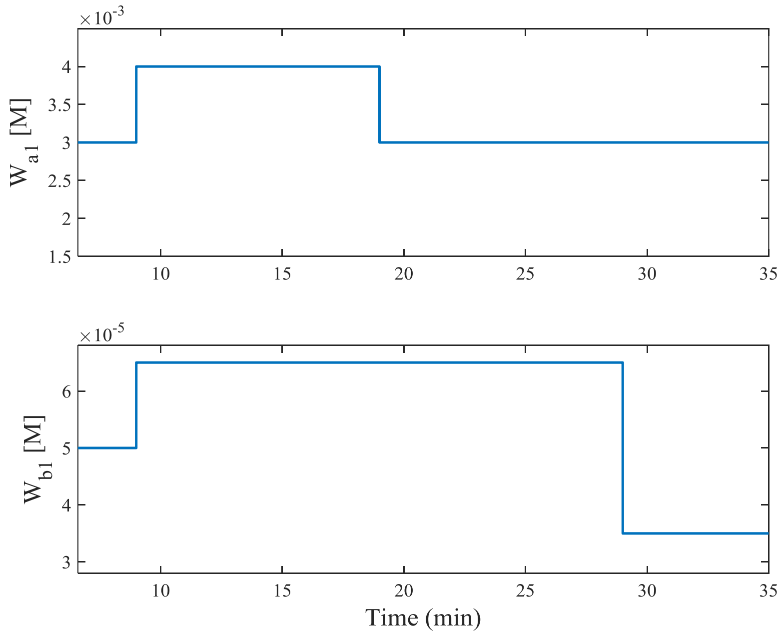

Figure 16. Disturbances in the reaction invariants of the acid stream in Scenario 4.

The control performance in Scenario 4 is shown in Figure 17. The fuzzy-PI controller clearly outperforms the conventional PI in both setpoint tracking and disturbance rejection cases. This is the result of the gain scheduling in the fuzzy-PI controller, as shown in Figure 18. Interestingly, the fuzzy logic makes timely changes to the controller gains in order to track the setpoint successfully. It temporarily reduces the integral gain K_I upon a setpoint change to weaken the integral action, which would otherwise cause oscillations or instability if overused [35], especially in the presence of measurement delay. At the same time, the fuzzy logic increases the proportional gain K_P to speed up the setpoint tracking. This adjustment of the controller gains in arbitrary directions is not possible with a single-parameter scheduling algorithm such as that proposed in [30].

A closer look into the fuzzy inference system reveals that 22 of 49 rules are fired at least once during the simulation. The unused rules would still be needed in other scenarios to ensure enough resolution for the fuzzification of the error and its derivative. Moreover, the maximum number of rules fired simultaneously in a single time step is 6.

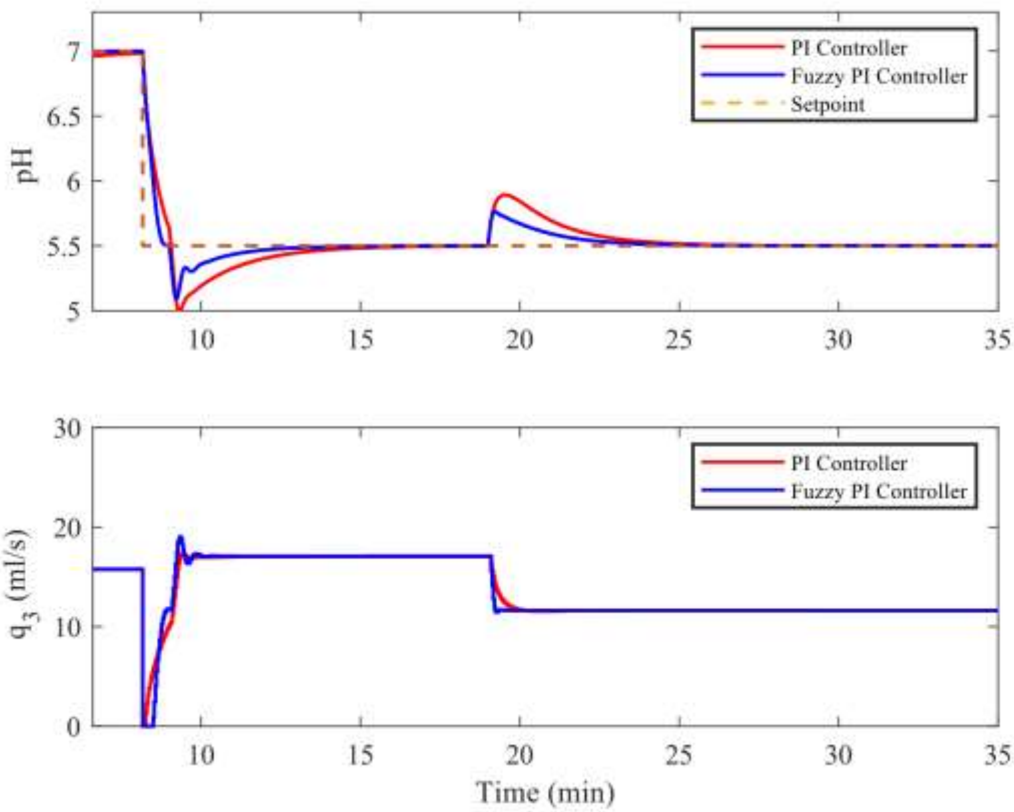

Figure 17. Comparing conventional and fuzzy-PI controllers in Scenario 4.

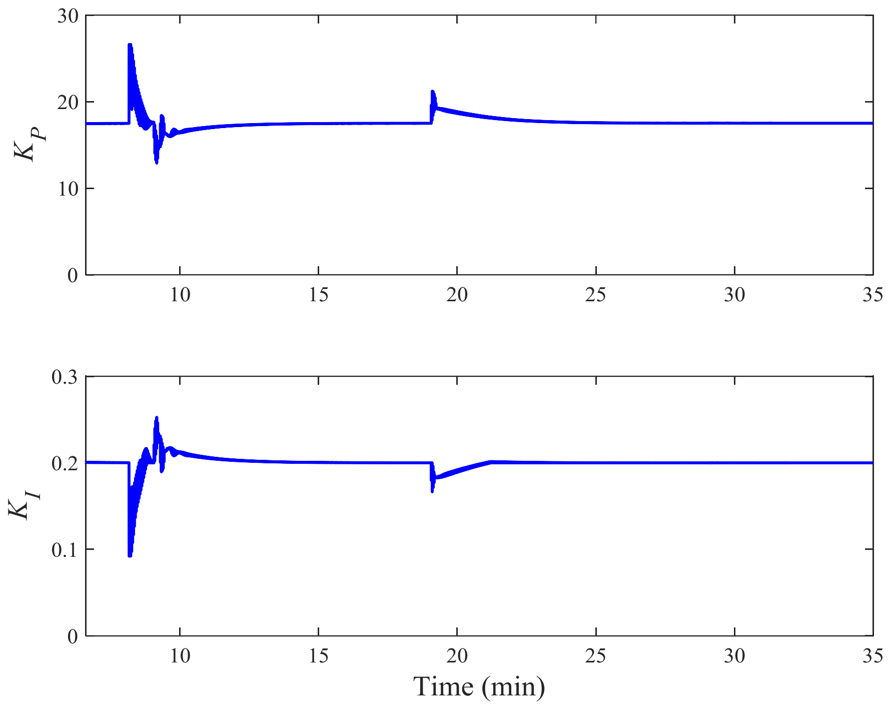

Figure 18. Proportional and integral gains varied by fuzzy logic in Scenario 4.

In Scenario 5, the pH setpoint is changed from 7 to 6, and then, from 7 to 7.5 (see Figure 19), while concentration disturbances are still present according to Figure 16.

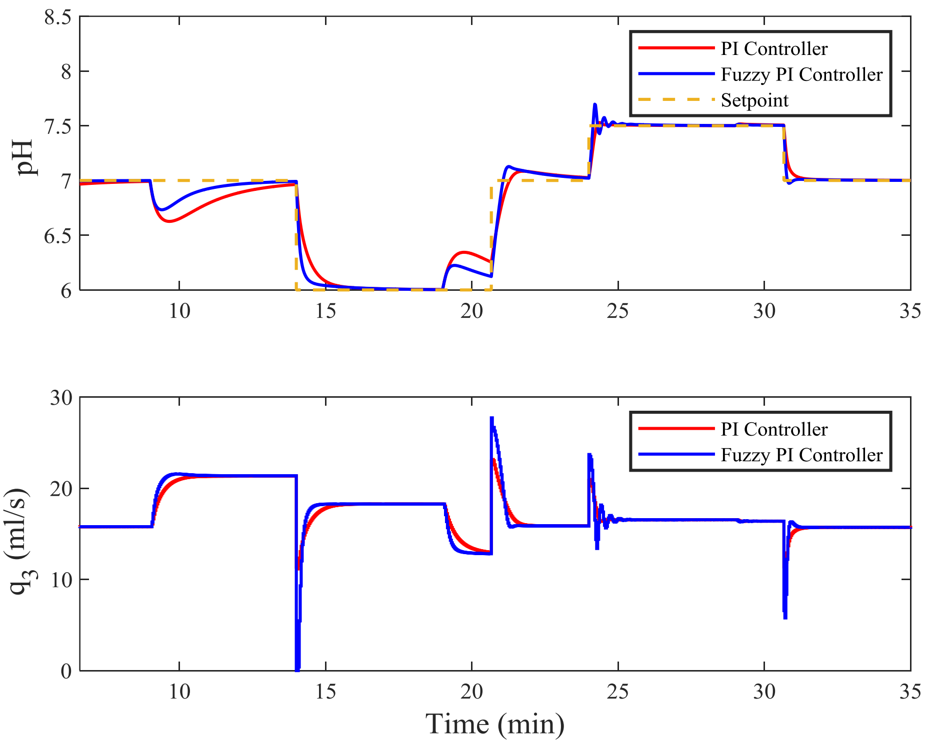

Figure 19. Comparing conventional and fuzzy-PI controllers in Scenario 5.

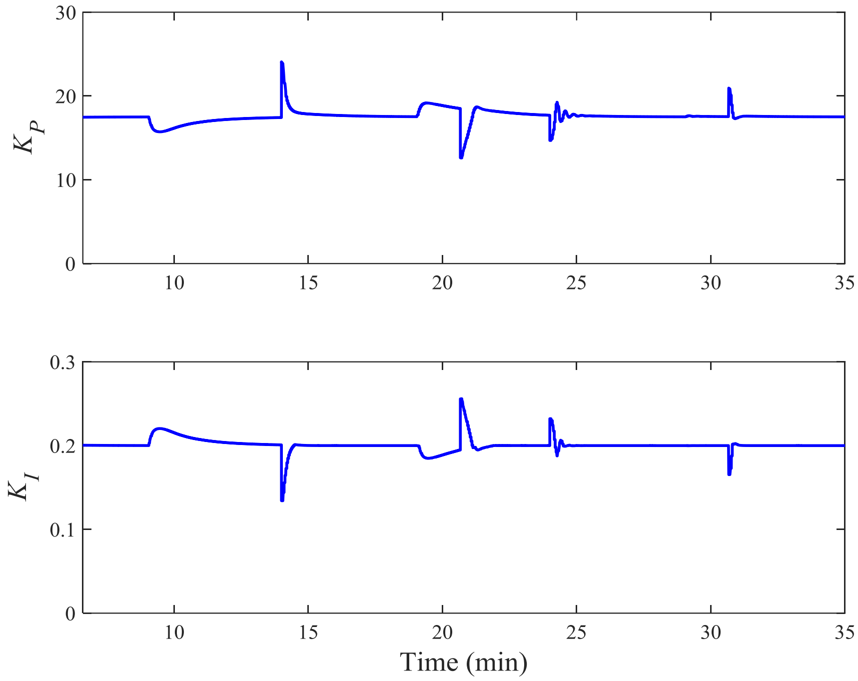

Figure 20. Proportional and integral gains varied by fuzzy logic in Scenario 5.

It is observed from Figure 19 that both controllers track the setpoint satisfactorily. However, the fuzzy-PI controller results in less deviation owing to the online adjustments of the proportional and integral gains, as shown in Figure 20.

4.4. Disturbance rejection with measurement noise

To evaluate controller robustness and industrial applicability, Scenario 6 is built with disturbances in the feed reaction invariants (Figure 21) and measurement noise. A low-pass filter with a time constant of 0.1 min was used to condition the noisy signal.

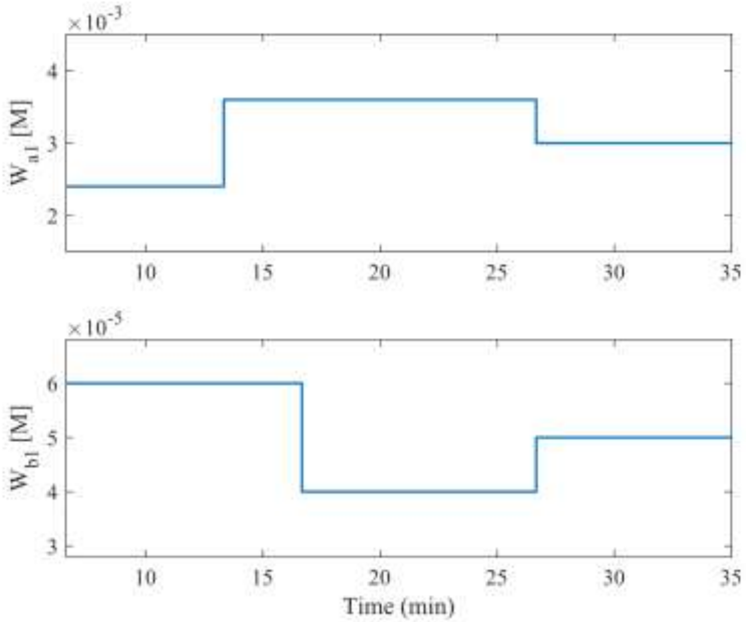

Figure 21. Disturbances in reaction invariants of the acid stream in Scenario 6.

As shown in Figure 22, the fuzzy-PI controller visibly maintains tighter setpoint tracking compared with the conventional PI.

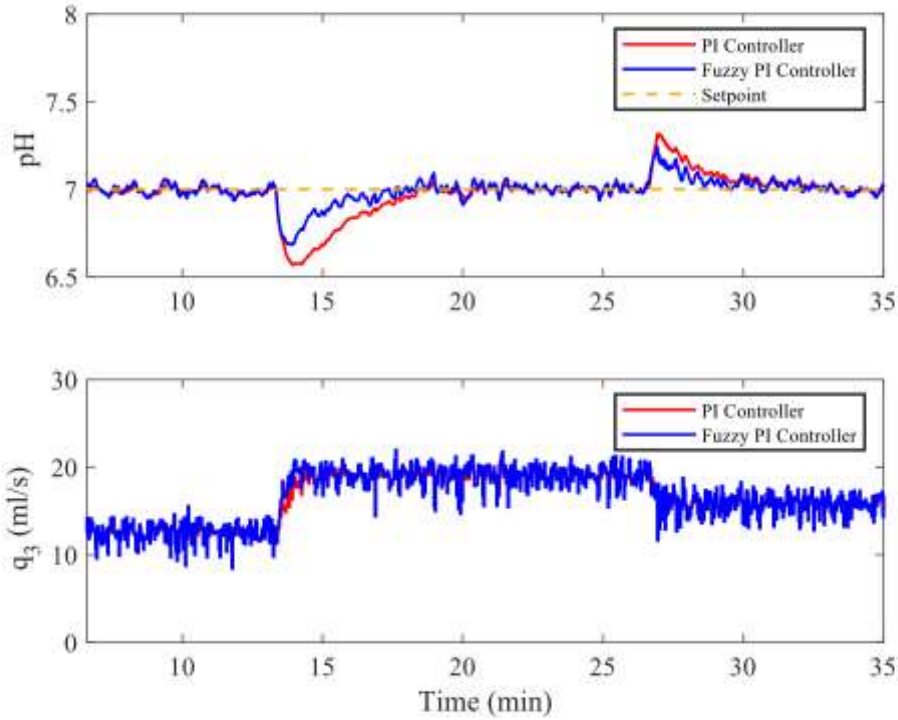

Figure 22. Comparing conventional and fuzzy-PI controllers in Scenario 6.

The proportional and integral gains for the fuzzy-PI controller in Scenario 6 are shown in Figure 23. It can be observed that the dynamic gain scheduling effectively compensates for and rejects the combined effects of measurement noise and disturbances.

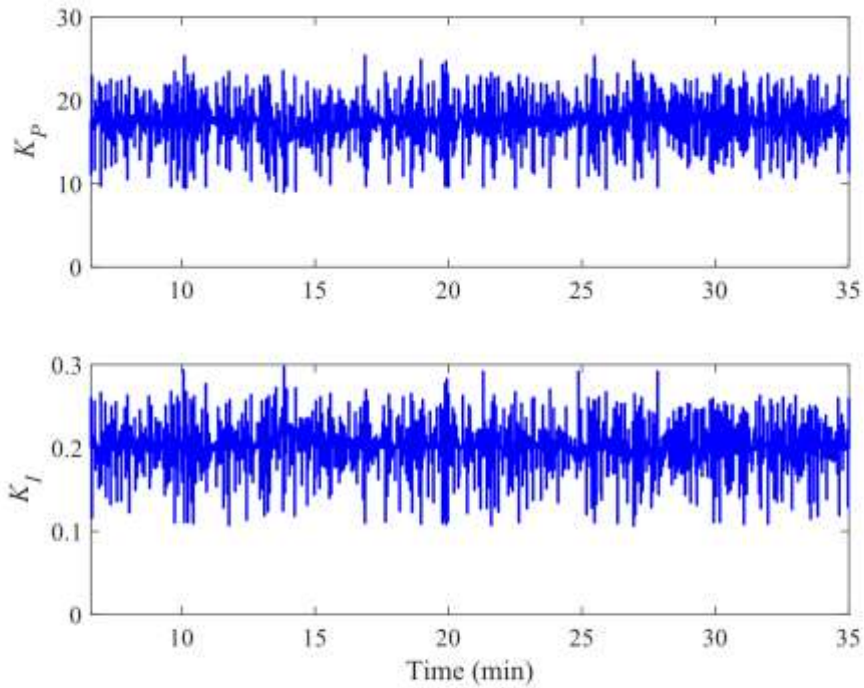

Figure 23. Proportional and integral gains varied by fuzzy logic in Scenario 6.

Finally, to compare the performance of the two controllers in all the scenarios quantitatively, the Integral of Absolute Error (IAE) is used as

$$IAE = \int_0^{T_f} |y_{sp} - y_m| dt \, . \quad (16)$$

The IAE results are presented in Table 3. In the first scenario, the conventional PI slightly outperforms the proposed fuzzy PI. The setpoint change in Scenario 1 is small. Therefore, the process remains close to its nominal point, for which the conventional PI is tuned. Consequently, the fuzzy PI would perform equally well or slightly worse as in this case. However, using a fixed-gain PI close to the nominal point may not be worth the small improvement and could even risk instability if an unmeasured disturbance suddenly occurs in that duration. As the scenarios include more severe changes or disturbances, the fuzzy PI significantly outperforms the conventional one, with the largest improvement seen in Scenario 4. This demonstrates the superiority of the proposed fuzzy gain scheduling in successfully handling measurement delay and process nonlinearity away from the nominal operating conditions.

Table 3.
IAE index for all the scenarios.

Scenario	Fuzzy-PI	Conventional PI	Improvement (%)
1	0.0812	0.0769	-5.59
2	0.4014	0.5938	32.40
3	2.2733	4.4705	49.15
4	1.3291	2.9194	54.47
5	1.4240	2.3090	38.33
6	1.2208	1.8574	34.27

5. Conclusion

A fuzzy gain scheduling digital PI controller was developed for a pH process in this study. The fuzzy system works with the control error and its derivative as the inputs and produces the proportional and integral gains in real time. Despite the fact that no effort for the systematic optimization of the membership functions was made, the resulting fuzzy-PI controller was shown to outperform a tuned conventional PI. This was specially true in the case of concentration disturbances, where the nonlinear process deviated significantly from the nominal point. The fuzzy-PI controller successfully maintained robust performance through timely reduction of the integral gain during rapid transients. The proposed fuzzy-PI approach can be applied to other pH processes with significantly different operating points. However, the fuzzy membership functions and defuzzification ranges may require re-tuning or re-design due to the system’s inherent nonlinearity. Future work can focus on improving the fuzzy system by considering additional inputs such as measurement delays and measured disturbances in order to enable proper control over a wider range of setpoints and disturbance scenarios.

Nomenclature

V	Volume of the tank, ml
K_{ai}	ith dissociation constant of the acid
K_{bi}	ith dissociation constant of the base
K_w	Dissociation constant of water
q_1	Flow rate of HNO_3, ml/s
q_2	Flow rate of NaOH, ml/s
q_3	Flow rate of $NaHCO_3$, ml/s
pH	$-\log([H^+])$
W_{ai}	Reaction invariant related to charge balance, M
W_{bi}	Reaction invariant related to carbonate species concentration, M
pK_{ai}	$-\log(K_{ai})$
pK_{bi}	$-\log(K_{bi})$
K_P	Proportional gain of the PI controller, ml/s
K_I	Integral gain of the PI controller, ml/s
$\overline{K_P}(k)$	Smoothed proportional gain, ml/s
$\overline{K_I}(k)$	Smoothed integral gain, ml/s
$e(k)$	Current control error
$\Delta e(k)$	Change in error at time k
$e(k-1)$	Previous control error
Δt	Sampling time, min
$p(k)$	Control signal at time step k, ml/s
$p(k-1)$	Control signal at the previous time step, ml/s
q	damping coefficient
y_{sp}	Setpoint value (desired pH)
y_m	Measured pH value with a delay

Appendix

Table A1. Nominal Operating Conditions for the pH Process.

Parameter	Value
V	2900 ml
K_{a1}	4.47×10^{-7}
K_{a2}	5.62×10^{-11}
K_w	1.00×10^{-14}
$[q_1]$	0.003 M HNO_3
	5.0×10^{-5} M H_2CO_3
$[q_2]$	0.03 M $NaHCO_3$
$[q_3]$	0.003 M NaOH
	5.0×10^{-5} M $NaHCO_3$
q_1	16.6 ml/s
q_2	0.55 ml/s
q_3	15.8 ml/s
pH	7.00
W_{a1}	0.003 M
W_{b1}	5.00×10^{-5} M
W_{a2}	-0.03 M
W_{b2}	0.03 M
W_{a3}	-3.05×10^{-3} M
W_{b3}	5.00×10^{-5} M
W_{a4}	-4.50×10^{-4} M
W_{b4}	5.50×10^{-4} M

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